THE INTERNAL ROTATION OF THE SUN

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Abstract  Helioseismology has transformed our knowledge of the Sun’s rotation.
Earlier studies revealed the Sun’s surface rotation, but now a detailed observational
picture has been built up of the internal rotation of our nearest star. Unlike the predictions
of stellar-evolution models, the radiative interior is found to rotate roughly uniformly.
The rotation within the convection zone is also very different from prior expectations,
which had been that the rotation rate would depend primarily on the distance from
the rotation axis. Layers of rotational shear have been discovered at the base of the
convection zone and in the subphotospheric layers. Studies of the time variation of
rotation have uncovered zonal-flow bands, extending through a substantial fraction of
the convection zone, which migrate over the course of the solar cycle, and there are
hints of other temporal variations and of a jet-like structure. At the same time, build-
ning on earlier work with mean-field models, researchers have made great progress in
supercomputer simulations of the intricate interplay between turbulent convection and
rotation in the Sun’s interior. Such studies are beginning to transform our understanding
of how rotation organizes itself in a stellar interior.

1. INTRODUCTION

Rotation is ubiquitous in astrophysical systems. During a system’s formation as
a result of gravitational contraction, its moment of inertia is reduced, sometimes
by many orders of magnitude; typically, therefore, any initial rotation is greatly

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enhanced. This principle appears to apply over a vast range of scales, from galaxies to neutron stars. Often the evolution of rotation seems closely linked to the evolution of the magnetic field, which, being frozen to the partly conducting plasma, is also amplified by contraction. Furthermore, a combination of rotational shear and turbulence may lead to dynamo effects, in stars as well as galaxies, which amplify the field. An understanding of these processes is essential for a description of most astrophysical phenomena. Yet, the details of the evolution of rotation and magnetic fields are poorly understood from a theoretical point of view, leading to obvious difficulties in the modeling of the systems, and observations rarely provide the necessary detailed insight.

1.1. Rotation and Dynamics of Stellar Interiors

Rotation plays an important role in the understanding of stellar formation and evolution (for a recent compilation on the general topic of stellar rotation, see Maeder & Eenens 2003). Stars are born from contraction of interstellar gas clouds, imparting rapid rotation to the protostar. The infalling material tends to be organized in an accretion disk, which, probably through the action of magnetic fields, often leads to the formation of collimated jets. From such disks, evidence for which has been found in many young stars, planetary systems may subsequently form. The rotation rate of the newly formed star is established through the interaction, likely of magnetic origin, between the protostar and the disk. Observations of young stellar clusters, whose members can be assumed to have the same age, show that the result is a wide range of rotation rates. The subsequent evolution of the rotation of a star is determined by the interplay between the possible loss of angular momentum in a stellar wind and the redistribution of angular momentum in the stellar interior. For stars of relatively low mass, including the Sun, angular-momentum loss takes place through a magnetically dominated stellar wind linked to the outer convection zone. The strength of the wind is related to the magnetic activity of the star, which in turn is generally assumed to be caused by dynamo processes within or just beneath the convection zone, presumably depending on the rotation rate of the star. Thus, there is a close coupling between rotation and the angular-momentum loss. This leads to the Skumanich law, according to which the surface angular velocity decreases gradually with age $t$ as $t^{-1/2}$ (Skumanich 1972; for a recent update on the observational evidence, see Barnes 2001). The redistribution of angular momentum by convection is a rapid process; in the solar case, at least, this has led to differential rotation, with the equator spinning more rapidly than the poles. In contrast, the loss of angular momentum through a stellar wind to spin down the star proceeds typically much more slowly. An understanding of angular-momentum transport in radiative regions, and hence predictions of the resulting rotation profile there and its coupling to convective regions, is so far rather uncertain.

Rotation has a dynamical effect on stellar structure and evolution through the modification by the centrifugal acceleration of hydrostatic balance. In most cases, however, this effect is modest. Much more important effects arise from the
redistribution of chemical elements that may be caused by circulation and instabilities induced by rotation. In particular, the nonuniform heating along equipotentials, induced by rotation, drives the large-scale Eddington-Sweet circulation (e.g., Sweet 1950). Indeed, without such effects, the surface composition of many stars, possessing thin outer convection zones, would be drastically modified by settling and selective radiative levitation. Modifications of this nature are seen in a few so-called ‘peculiar’ A and B stars, which indicates that little mixing is taking place. However, in the ‘normal’ majority of stars, mixing—likely caused by rotation—must overwhelm the effects of settling and levitation. Rotationally induced mixing processes in stellar interiors also appear to account for several cases of surface-abundance changes during stellar evolution, as observed, for example, in massive stars or in red giants (for a review, see Maeder & Meynet 2000). Mixing may also affect a star’s evolution by changing its composition structure. Thus, an understanding of stellar rotation is crucial for the modeling of stellar structure and evolution. Indeed, as discussed in Section 7.3, rotationally induced mixing may be important in the Sun. A general review of mixing in stars was given by Pinsonneault (1997).

The Sun exhibits magnetic activity that varies cyclically, with a period of approximately 22 years. (The number of sunspots changes with a period of 11 years, whereas the magnetic polarity has a period of 22 years.) This is most evident in the presence on the solar surface of sunspots but is reflected in many other aspects of the solar atmosphere, such as the distribution of active regions and the structure of the corona. Similar magnetic activity has been inferred in other stars through a variety of observations; in many cases, cyclic variations similar to that observed in the Sun are seen. (For a general overview of solar and stellar magnetic activity, see Schrijver & Zwaan 2000.) It is generally assumed that underlying the magnetic activity is some form of dynamo action, involving interaction between stellar rotation and convective motion. Support for this comes from observed correlations between various measures of activity and properties of stellar rotation and convection (e.g., Hartmann & Noyes 1987, Schrijver & Zwaan 2000). It would follow, therefore, that an understanding of stellar magnetic activity is dependent on information about the interior rotational properties of stars. Dynamo action is similarly believed to be responsible for magnetic fields of various objects ranging from galaxies (e.g., Beck et al. 1996) to accretion disks (e.g., Vishniac & Brandenburg 1997) and planets (Glatzmaier & Roberts 1997, Proctor & Gilbert 1994).

Rotation plays important roles in other areas of solar and stellar astrophysics. A rapidly rotating solar core, not obviously inconsistent with the general understanding of the evolution of solar rotation, was invoked in early attempts to explain the observed solar neutrino deficit (e.g., Bartenwerfer 1973, Demarque, Mengel & Sweigart 1973). Furthermore, rapid interior rotation could distort the Sun’s external gravitational field and hence affect tests of gravitational theories based on observations of planetary motion (e.g., Dicke 1964); the resulting oblateness of the Sun’s surface might be difficult to discern among other latitudinal variations in the Sun’s surface properties.
Observational data on stellar rotation were, until fairly recently, limited to determinations of the solar-surface rotation as a function of latitude and to measurements of general rotation rates, or projected rotational velocities, of distant stars. However, over the past two decades, our knowledge of the solar interior has been revolutionized through helioseismology, i.e., the analysis of observed oscillations of the solar surface. In particular, rotation causes a splitting of the frequencies of oscillations, from which properties of the internal rotation can be inferred. Here, we review information on the Sun’s internal rotation and other aspects of its internal dynamics obtained from helioseismology and other sources, and we discuss the current understanding of these results from theoretical considerations and numerical modeling. Previous reviews of solar rotation were given in these volumes by Gilman (1974) and Howard (1984), the latter immediately before the first extensive helioseismic results on the rotation of the solar interior were presented by Duvall et al. (1984). Among the many general reviews on helioseismology, we mention those by Gough & Toomre (1991) and Christensen-Dalsgaard (2002), whereas Thompson et al. (1996) and Schou et al. (1998) discussed helioseismic inference of solar rotation. Beck (2000) provided a detailed review of surface and helioseismic observations of solar rotation. Miesch (2000) reviewed the numerical modeling of solar convection and rotation.

1.2. Stellar Rotation

Several techniques are available to investigate the rotation of distant stars. Rotation, when sufficiently fast, causes a measurable broadening of the spectral lines. This provides a measure of the line-of-sight component of the rotational velocity \( v_{\text{rot}} \), i.e., \( v_{\text{rot}} \sin i \), where \( i \) is the inclination angle between the rotation axis and the line of sight. With a sufficient number of stars in an otherwise uniform sample, it might be possible to correct for the inclination angle in a statistical sense, assuming random orientation of the rotation axes. In practice, modern observations allow measurement of \( v_{\text{rot}} \sin i \) as low as \( 5 \) km s\(^{-1}\).

A direct determination of the rotation period is possible when the star displays an inhomogeneous surface, resulting in observable periodic variations as the star rotates. This is typically the case for lower main-sequence stars that show solar-like activity. If sufficiently large starspots are present, broadband photometry can show the rotational modulation as demonstrated, for example, by observations of stars in the Pleiades cluster (Krishnamurthi et al. 1998). Alternatively, other activity measures may be observed. In an extensive program to study activity cycles in other stars (e.g., Wilson 1978), fluxes in the calcium H and K lines have been observed; these fluxes are measures of chromospheric activity that also show rotational modulation (e.g., Donahue, Saar & Baliunas 1996).

Finally, observations of rotational splittings of stellar oscillation frequencies would evidently provide measures of stellar rotation. So far, this technique has only been successfully applied in the case of white dwarfs (e.g., Winget et al. 1994). However, it holds great promise for a broad range of stars, including
those with solar-like oscillations, with the advent of planned asteroseismic space projects.

The stellar data generally indicate that, at least for lower main-sequence stars, rotation tends to slow down with increasing age, in accordance with the general picture of angular-momentum loss through a magnetically coupled stellar wind. The evolution of rotation with stellar age can be inferred from observations of stellar clusters of different ages, as demonstrated in Figure 1.

In the Pleiades, a relatively young cluster, the large scatter in rotational velocity presumably reflects differences in the formation history of the stars. However, by the age of the Hyades, rotation has settled down to an essentially unique dependence on the stellar effective temperature, here measured by the color index.

Evidence for differential rotation in stars other than the Sun has been found. Donahue, Saar & Baliunas (1996) found that the rotational periods inferred from the calcium H and K variations showed changes with time, which they interpreted as a reflection of variations in time of the latitudes of the active regions that dominated the signal. This could be the case, for example, if the emergence of active regions showed a latitude variation similar to what is observed in the solar butterfly diagram. In fact, by applying the same technique to solar data as that used for the distant stars, Donahue, Saar & Baliunas (1996) were able to recover the solar differential rotation over the latitude band spanned by the solar active regions. Also, Reiners & Schmitt (2002a,b) showed that differential rotation can be inferred from Fourier transform analysis of rotationally broadened spectral lines, provided that data of sufficient quality and with sufficiently high spectral resolution are available.

1.3. Solar-Surface Rotation

Detailed observations have been carried out of the solar-surface rotation by tracking the motion of surface features such as sunspots and, more recently, by using
Doppler-velocity measurements. It was firmly established by the nineteenth century, through careful tracking of sunspots at different latitudes on the Sun’s surface, that the Sun is not rotating as a solid body, but rather shows surface differential rotation with latitude: at the equator, the rotation period is approximately 25 days, but it increases gradually toward the poles where the period is estimated to be approximately 36 days. This must somehow result from the redistribution of angular momentum by motions within the convection zone. The latitude variation is traditionally parametrized as an expansion in \( \cos^2 \theta \), where \( \theta \) is co-latitude (e.g., Snodgrass 1983). The Doppler rotation rate, for example, is approximated by

\[
\frac{\Omega}{2\pi} = (451.5 - 65.3 \cos^2 \theta - 66.7 \cos^4 \theta) \text{ nHz}
\]

(Ulrich et al. 1988). Examples of such surface-rotation fits are shown in Figure 2. The difference between the Doppler and magnetic-feature rotation rates may indicate that the magnetic features are anchored at some depth beneath the surface, where rotation might be slightly more rapid. The anomalously high rate of the Doppler features apparently does not correspond to any material motion in the Sun. Gizon, Duvall & Schou (2003) have argued that the supergranular cells, which dominate the Doppler-feature signal, appear to have wave-like features.

![Figure 2](https://example.com/figure2.png)

**Figure 2** Near-surface solar-rotation rates as determined from surface spectroscopic Doppler-velocity measurements (*solid line*), tracking magnetic features (*dashed line*), tracking sunspots, *dot-dashed line* from magnetogram correlation tracking), and tracking Doppler features resulting from large-scale (supergranular) convective flow patterns (*dotted line*); the curves correspond to fits of the form given in Equation 1, with coefficients adapted from Beck (2000). A rotation rate of 320 nHz corresponds to a period of 36 days, and a rate of 460 nHz corresponds to a period of 25 days.
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properties; thus, the observed rate may reflect the propagation speed of these wave-like patterns.

For at least one century, surface differential rotation of the Sun has not changed by more than approximately 5% of the equatorial value (Gilman 1974, Howard 1984, Rüdiger 1989). However, temporal variations on the order of a few percent have been detected. Doppler-velocity measurements of the Sun’s surface rotation reveal narrow bands of weakly slower and faster rotation, superimposed on the rather smoothly varying rotation rate as a function of latitude. These variations, referred to as torsional oscillations, migrate toward the equator on a timescale comparable to the solar activity cycle (Howard & LaBonte 1980). Ulrich (2001) recently provided an extensive analysis of these and other related flows that also emphasized the close relation between the latitudes of the flows and the dominant magnetic activity. In addition, Doppler observations reveal large-scale nonrotational flows on the solar surface; these include flows from the equator toward the pole which may represent meridional circulation in the convection zone (e.g., Hathaway 1996, Hathaway et al. 1996).

1.4. Solar Internal Rotation

The properties of the solar interior have been probed in great detail from observations of oscillations of the solar surface. Most oscillation modes are predominantly standing acoustic waves, whereas others, at relatively short wavelengths, are essentially surface-gravity waves. Analyses of their frequencies have yielded stringent constraints on models of solar hydrostatic structure. Furthermore, the observations have provided inferences of the Sun’s angular velocity as a function of latitude and depth over most of the solar interior. An example of the results on rotation is shown in Figure 3. Throughout the convection zone, the rotation rate shows a variation with latitude similar to that of the surface rotation, although with significant subtler details. Important among these is the near-surface shear layer, wherein the rotation increases with depth (at least at low latitudes) in the outer 5%. The radiative interior, on the other hand, rotates at a nearly uniform rate, intermediate between the surface-equatorial and high-latitude rates. The transition between these two different regimes takes place in a narrow region, the tachocline, where there is evidently a strong radial rotational shear. This region is generally believed to play an important role in the generation of the large-scale solar magnetic field (e.g., Weiss 1994).

The rotational profile shown in Figure 3 raises a number of theoretical questions. Simple theoretical considerations and early numerical models suggested that rotation in the convection zone should be “constant on cylinders,” with the angular velocity depending predominantly on the distance from the rotation axis; this is manifestly not satisfied in the Sun. Thus, there must be processes that redistribute angular momentum in the convection zone, resulting in the observed profile. Furthermore, if angular momentum has been lost from the solar surface in the solar wind, mechanisms are required to slow down the solar core, and in the process establish and maintain the sharp gradients in the tachocline.
2. HELIOSEISMIC PROBING OF THE SOLAR INTERIOR

The Sun oscillates simultaneously in millions of modes, with periods typically between approximately 3 and 15 min; the horizontal scales of the modes range from radial oscillations, where the Sun preserves its spherical shape as it oscillates, to modes with a local horizontal wavelength of only a few thousand kilometers on the solar surface. The amplitude of an individual mode, as observed in Doppler velocity, is at most approximately 20 cm s\(^{-1}\), or a few parts per million in intensity. Modes with amplitudes as low as 3 mm s\(^{-1}\) have been detected (García et al. 2001). The detection of such small effects in the Sun’s turbulent atmosphere is only possible because the modes are coherent over the Sun’s surface, and over timescales of weeks or months.

The detailed observations of solar oscillations have required the development of extremely stable dedicated instrumentation deployed in large coordinated projects. Long-running efforts include the ground-based BiSON (Birmingham Solar Oscillation Network) (Chaplin et al. 1996) and IRIS (International Research on the Interior of the Sun) (Fossat 1991) networks, for observation of disk-averaged Doppler velocity. Results of major importance for our understanding of the dynamics of the solar convection zone have been obtained from the GONG (Global Oscillation Network Group) (Harvey et al. 1996) network as well as from the MDI (Michelson Doppler Imager) (Scherrer et al. 1995) instrument on the SOHO (Solar and Heliospheric Observatory) spacecraft. SOHO also carries the GOLF (Global Oscillations at Low Frequency) (Gabriel et al. 1995) and VIRGO (Variability of Solar Irradiance and Gravity Oscillations) (Fröhlich et al. 1995) instruments, which emphasize large-scale modes at low frequency.

2.1. Basic Aspects of Helioseismology

The observed solar oscillations can be described in terms of the global, or normal, modes of the Sun. Specifically, the behavior of a mode is described by a spherical harmonic

\[ Y_l^m(\theta, \phi) = c_{lm} P_l^m(\cos \theta) \exp(im\phi) \]

as function of co-latitude \(\theta\) and longitude \(\phi\); here, \(P_l^m\) is a Legendre function, and \(c_{lm}\) is a normalization constant. The spherical harmonics are characterized by two integer numbers, their degree \(l\) and their azimuthal order \(m\). In addition, a mode is identified by its radial order, \(n\), which is given approximately as the number of nodes in the radial direction. The dependence on time \(t\) of an oscillation with a given \(n, l, \text{ and } m\) can be written as \(\exp(-i\omega_{nlm}t)\), where \(\omega_{nlm}\), which, as indicated, depends on all three characteristic numbers, is the angular frequency. The mean multiplet frequency \(\omega_{nl}\), obtained as an average over \(m\) of \(\omega_{nlm}\), is approximately determined by the spherically symmetric component of the stellar structure as characterized, for example, by the variation of the sound speed \(c\) and density \(\rho\) with distance \(r\) to
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Different modes are sensitive to different regions of the Sun, depending on their frequency, degree, and azimuthal order. The modes are confined between latitudes $\pm \cos^{-1}(m/L)$, where $L = l + 1/2$. Thus modes with $m = l$ reside in a narrow region near the equator, whereas modes of low $m$ extend to near the poles. Most of the observed modes are essentially standing acoustic waves ($p$ modes). In the radial direction, these are largely confined outside an inner turning point at the distance $r = r_t$ from the center, where the horizontal phase speed $\omega_{nlm} r / L$ of the mode equals the local sound speed $c(r)$. Thus, low-degree modes extend over much of the Sun, whereas high-degree modes are confined close to the solar surface. In addition, at moderate and high degree, one observes $f$ modes, which correspond to $n = 0$ and have the nature of surface-gravity waves. For $f$ modes, the displacement varies with $r$ as $\exp(-k z)$, where $z = R - r$ is depth, $k = L / R$, and $R$ is the surface radius; these modes also become increasingly confined near the surface with increasing degree. By exploiting the different sensitivities of the modes, helioseismology is able to make inferences about localized conditions inside the Sun.

By applying such techniques to the multiplet frequencies $\omega_{nlm}$, one may constrain the radial structure of the Sun by determining corrections to the sound speed and density of solar models. Such analyses show that current solar models are good approximations to the actual solar structure (e.g., Basu et al. 1997, Gough et al. 1996, Turck-Chièze et al. 2001).

2.1.1. ROTATIONAL SPLITTING OF FREQUENCIES

For the present purpose, the most important aspect of the solar modes is their sensitivity to solar rotation, which affects the frequencies through advection and, to a lesser degree, the Coriolis force. Solar rotation is sufficiently slow that higher-order effects of rotation can be neglected to a good approximation. The effect can be quantified in terms of a weight function, or kernel, such that the rotational splitting $\delta \omega_{nlm}$ is obtained as

$$
\delta \omega_{nlm} \equiv \omega_{nlm} - \omega_{nl0} = m \int_0^\pi \int_0^\pi K_{nlm}(r, \theta) \Omega(r, \theta) r \, dr \, d\theta.
$$

The kernels $K_{nlm}$ can be calculated from the eigenfunctions for a nonrotating model (Schou, Christensen-Dalsgaard & Thompson 1994); given the helioseismic constraints on the structure of solar models, the kernels can, to a very good approximation, be taken as known. It should be noted that the kernels depend on $m^2$, not $m$, so that the rotational splitting $\omega_{nlm} - \omega_{nl0}$ is an odd function of $m$. Also, the kernels are symmetrical around the equator. As a result, the rotational splitting is only sensitive to the component of $\Omega$ that is similarly symmetrical. As described in Section 2.2, the data can also be given in the form of so-called $a$ coefficients: these too can be related to the underlying rotation rate $\Omega$ by kernels, in a form similar to Equation 3.

The kernels vary from mode to mode, reflecting the regions of confinement. A few examples are illustrated in Figure 4. The observed modes include some that
Figure 4  Contour plots of rotational kernels \( K_{nlm} \) in a solar quadrant, with the equator as the horizontal axis. The modes all have frequencies near 2 mHz; the following pairs of \((l, m)\) are included: \((a) (5, 2); (b) (20, 8); (c) (20, 17); and (d) (20, 20). The dotted circle marks the location of the lower radial turning point \( r_t \), and the dotted line shows the latitudinal turning point at \( \sin^{-1} m/L \).

penetrate essentially to the solar center, others that are trapped very near the surface, and the whole range of intermediate penetration depths, with a similar variation in latitudinal extent. Thus, the observed frequency splittings provide a similarly wide range of averages of the internal rotation. However, the amount of information that is available depends greatly on the location in the Sun. Regions near the surface affect most of the observed modes, including modes of high degree where the large number of \( m \) values ensure good resolution in the latitude direction. In contrast, only a limited number of modes of the lowest degrees penetrate to the solar core, providing essentially no latitude resolution. Also, the sound speed is high in the solar core. Thus, the \( p \) modes, regarded as a superposition of propagating sound waves, spend little time in this region; consequently, the sensitivity is reduced. The effect of the core on the frequencies is further reduced by its small volume and the short distance to the center. The effect on the kernels is illustrated in Figure 5; for simplicity we consider kernels for spherically symmetric rotation, \( \bar{\tilde{\alpha}} = \tilde{\alpha}(r) \), where

\[
\omega_{nlm} = \omega_{nl0} + m \beta_{nl} \int_0^R K_{nl}(r) \Omega(r) \, dr.
\]

The kernels are normalized such that \( \int K_{nl}(r) \, dr = 1 \). It is particularly striking that, according to Figure 5b, only 10% of the splitting arises in the inner 30% of the radius for the modes considered.

2.1.2. INVERSIONS TO INFER THE ROTATION PROFILE  The analysis of the oscillation data to infer properties of the solar interior is often characterized as inversion (Gough & Thompson 1991). The goal of the analysis is to infer the properties of \( \Omega(r, \theta) \) from the data, taking into account also the observational errors from relations such as Equation 3. Ideally, we wish to determine an approximation \( \bar{\Omega} \) to the true angular velocity. In most cases considered so far, the analysis corresponds either implicitly or explicitly to making linear combinations of the data. Thus, to
infer $\bar{\Omega}$ at a point $(r_0, \theta_0)$, coefficients $c_i(r_0, \theta_0)$ are determined such that, as far as possible, $\bar{\Omega}(r_0, \theta_0) \approx \bar{\Omega}(r_0, \theta_0) = \sum c_i(r_0, \theta_0) \Delta_i$, where the label $i$ stands for the modes $(n, l, m)$, and $\Delta_i = m^{-1} \delta_{\theta \theta m}$. From Equation 3, it then follows that

$$\bar{\Omega}(r_0, \theta_0) = \int K(r_0, \theta_0; r, \theta) \Omega(r, \theta),$$  

where the averaging kernels $K$ are given by

$$K(r_0, \theta_0; r, \theta) = \sum c_i(r_0, \theta_0) K_i(r).$$  

(5)  

(6)

If the standard errors $\sigma(c_i)$ of the observations are known, the error in the inferred angular velocity $\bar{\Omega}(r_0, \theta_0)$ can be found, given the $c_i$.

It is evident from Equation 5 that the coefficients $c_i(r_0, \theta_0)$ should be determined such that $K(r_0, \theta_0; r, \theta)$ is localized near $(r, \theta) = (r_0, \theta_0)$. The extent of $K$ provides a measure of the resolution of the inversion. At the same time, it must be ensured that the error on $\bar{\Omega}(r_0, \theta_0)$ is reasonable. In general, there is a trade-off between error and resolution determined by one or more parameters of the procedure.

In one commonly used inversion procedure, the regularized least-squares (RLS, also known as Tikhonov regularization) procedure, the solution is parametrized and the parameters are determined through a least-squares fit of the data to the right side of Equation 3. To ensure that the solution is well behaved and the errors are of reasonable magnitude, the fit is regularized by limiting, for example, a measure of the square of the second derivative of the solution, thus penalizing rapid variations.
Figure 6 Averaging kernels in a solar quadrant for the RLS-inversion solution shown in Figure 7, targeted at the following radii and latitudes in the Sun: (a) $0.540R$, $60^\circ$; (b) $0.692R$, $0^\circ$; (c) $0.692R$, $60^\circ$; (d) $0.952R$, $60^\circ$. The corresponding locations are indicated with crosses. Positive contours are shown as solid lines, negative contours as dashed lines. (Adapted from Schou et al. 1998.)

(e.g., Craig & Brown 1986). From the results of the fit, the coefficients $c_i(r_0, \theta_0)$, and hence the averaging kernels, can be determined. In a second type of method, the optimally localized averages (or OLA) procedure, the $c_i(r_0, \theta_0)$ are determined explicitly to obtain a localized averaging kernel $K(r_0, \theta_0; r, \theta)$, while at the same time limiting the errors. This method has been implemented in a variant called Subtractive OLA (SOLA) (Pijpers & Thompson 1992, 1994). Schou et al. (1998) gave details of these procedures and their results. Illustrating the resolution of such inversions, Figure 6 shows selected averaging kernels for RLS inversion of data obtained with the MDI instrument; other inversion techniques have similar resolution.

As already remarked, the kernels (Figure 4) are symmetrical around the solar equator, and so we can infer only the similarly symmetric component of rotation. This must be kept in mind when interpreting the results. As discussed in Section 4, investigations that do not suffer from this constraint are possible with local helioseismology.

2.2. Data Analysis Procedures

The basic helioseismic data are time series of images, most often of the Doppler velocity as a function of position on the solar disk. These undergo spatial analysis to isolate the individual spherical harmonics, followed by Fourier analysis in time to determine the frequencies. These steps all involve a number of difficulties of potential serious significance for the final result. A particular challenge is the final step, often referred to as peak bagging, of determining the oscillation frequencies through fits to the Fourier spectra. These fits must take into account the statistical properties of the oscillations determined by the stochastic processes that excite and damp them, and they are affected by the incomplete separation between different spherical-harmonic components arising from observing less than half the solar surface. Techniques for the analysis of helioseismic data have been presented, for example, by Anderson et al. (1990), Schou (1992) and Hill et al. (1996).
Owing to the difficulties in isolating individual frequencies $\omega_{nlm}$, the fit is often carried out, or presented, in the form of the so-called $a$ coefficients in an expansion of the form

$$\omega_{nlm} = \omega_{n0} + 2\pi \sum_{j=1}^{\text{max}} a_j(n,l) P_j^l(m),$$

(7)

where the $P_j^l$ are orthogonal polynomials (e.g., Ritzwoller & Lavely 1991, Schou, Christensen-Dalsgaard & Thompson 1994). It follows from the properties of the rotational splitting (Equation 3), that rotation contributes only to the odd $a$ coefficients, i.e., those coefficients $a_{2k+1}$, $k = 0, \ldots$ of odd index. It is straightforward to show from Equation 3 that these coefficients are similarly linearly related to the angular velocity $\Omega(r, \theta)$ with kernels $K_{nl}^{(a)}(r, \theta)$; hence, they lead to an inverse problem similar to that given in Equation 3.

A particular type of spatial analysis is achieved in observations where light integrated over the solar disk is analyzed. Such observations are sensitive only to modes of the lowest degrees (in practice $l = 0–3$) (see, e.g., Christensen-Dalsgaard & Gough 1982, Dziembowski 1977). Furthermore, because the solar rotation axis is always close to the plane of the sky, only modes of even $l - m$ are observed as a result of the symmetry of the spherical harmonics.

Extensive sets of helioseismic data have become available over the past decade through major ground-based and space-based observational efforts. Disk-averaged observations from the BiSON (Chaplin et al. 2001) and IRIS (Fossat et al. 2003) ground-based networks and with the GOLF instrument on the SOHO spacecraft (Gelly et al. 2002) have yielded rotational splittings with reasonable accuracy for the lowest-degree modes. Data spanning low and moderate degrees have been obtained from the LOWL instrument (Tomczyk et al. 1995). The GONG network (Harvey et al. 1996) has yielded extensive data covering a broad range of modes, whereas an even more extensive mode set has been covered by the MDI instrument on SOHO (Rhodes et al. 1997, Scherrer et al. 1995). Very significantly, these detailed datasets now extend over a large fraction of, or an entire, 11-year solar-sunspot cycle, allowing studies of possible variations in solar internal structure and rotation with the solar cycle.

### 3. INTERNAL ROTATION DEDUCED FROM GLOBAL MODES

Early results on rotation near the solar equator were obtained by Duvall et al. (1984), from analysis of observations of sectoral modes by Duvall & Harvey (1984). These results showed little variation with $r$ in the rotation rate, although they gave some indication that the radiative interior rotates at a rate slightly below the equatorial surface value. With the first observations of the dependence on $m$ of the rotational splitting (Brown 1985, Duvall, Harvey & Pomerantz 1986), it became possible to constrain also the latitudinal variation of rotation.
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(e.g., Brown & Morrow 1987, Brown et al. 1989, Christensen-Dalsgaard & Schou 1988, Kosovichev 1988, Libbrecht 1988, Rhodes et al. 1990). The results indicate that the variation on the solar surface extends through the convection zone, whereas the radiative interior showed little variation in rotation with latitude. A rotation profile constant on cylinders in the convection zone could apparently be ruled out (e.g., Thompson 1990; see also Schou & Brown 1994). The transition between the latitude variation in the convection zone and the almost uniform rotation in the radiative interior was found to take place across a relatively thin region near the base of the convection zone (Dziembowski, Goode & Libbrecht 1989, Goode et al. 1991).

3.1. Rotation of the Convection Zone

Analyses of the extensive data obtained during the past decade have largely confirmed these early results, although greatly refining the details. To discuss the inferences within and just below the convection zone, we consider results obtained by Schou et al. (1998), some of which were already shown in Figure 3 (see also Thompson et al. 1996). Further details are shown in Figure 7, which includes an inversion using the RLS technique, and in the inversion in Figure 3, which used the SOLA method. The underlying dataset, obtained with the MDI instrument, allows inference of rotation in most of the solar interior with good resolution and modest errors; however, as indicated by the blank area in Figure 3, there are regions

![Figure 7](image)

**Figure 7** Inferred rotation rate $\Omega / 2\pi$ as a function of fractional radius $r / R$ at the latitudes indicated. The results in the outer part of the Sun, for $r \geq 0.5 R$, were obtained from inversion of 144 days of MDI data (Schou et al. 1998), whereas the results for $r \leq 0.45 R$ were obtained from a combination of LOWL and BiSON data (Chaplin et al. 1999). The dashed lines, with 1-$\sigma$ error bands, show the results of an RLS inversion. The symbols were obtained with a SOLA inversion; the vertical bars show 1-$\sigma$ errors, whereas the horizontal bars provide an indication of the resolution of the inversion, as determined by the widths of the averaging kernels. Note that the result becomes much more uncertain in the deep interior, where, furthermore, the different latitudes cannot be separated.
where no reliable results can be obtained. The high latitudes are sampled only by a few modes with low $m$, and, as a result of the small distance to the axis, rotation of this region has little effect on the rotational splitting. We remark parenthetically that, because of the global nature of the modes, helioseismic determinations of the rotation rate can nonetheless reach to higher latitudes than can reliably be measured in direct Doppler measurements of surface rotation (see, for example, Ulrich 2001). In addition, determination of rotation of the deep interior requires accurate data on low-degree modes, for which the MDI instrument was not optimized.

Although the overall tendency is for rotation to be roughly independent of $r$ within the convection zone except at high latitude, interesting variations are clearly visible. A striking effect is the near-surface shear layer in the outer approximately 5% of the solar radius, in which rotation increases with depth at least at low latitude. Deubner, Ulrich & Rhodes (1979) already noticed this trend in one of the very early helioseismic analyses of solar rotation based on observations of high-degree modes. These results confirmed even earlier evidence for the existence of a near-surface shear layer based on variations among the different measures of surface-rotation rate evident in Figure 2: if the magnetic tracers of rotation were anchored at some depth beneath the surface, they would reflect the faster rotation rate found there (Foukal & Jokipii 1975, Deubner, Ulrich & Rhodes 1979, Korzennik et al. 1990). Corbard & Thompson (2002) carried out a detailed analysis of the shear layer on the basis of $f$-mode data and found that at low latitudes, below approximately 30°, the logarithmic derivative $d \log \tilde{\Omega}/d \log r \simeq -1$, whereas at higher latitudes, the rate of shear decreases in magnitude, possibly changing sign at a latitude of approximately 50°.

At high latitudes, the rotation shows a more complex behavior. Near the pole, the rotation rate decreases sharply relative to a three-term fit to results obtained at lower latitudes of the form given in Equation 1 (Birch & Kosovichev 1998, Schou et al. 1998). Finer structures may also exist. These include a local enhancement of the rotation rate, i.e., a possible jet-like structure, at a latitude of 75°, visible in the cut at this latitude in Figure 7. However, it should be noted that, as discussed by Howe et al. (1998), data from the GONG project, and a different analysis of the MDI time series, fail to show such a jet. Indeed, as is evident from Figure 7, there are substantial differences between the results of the inferences obtained at high latitudes with the different RLS and SOLA inversion techniques, whereas the results at lower latitude are largely consistent. Schou et al. (2002) discuss in detail these and other systematic differences between datasets and analysis methods. These differences should evidently be understood before detailed conclusions are drawn from the subtle features of the results.

### 3.2. Tachocline

From a dynamical perspective, perhaps the most interesting region is the tachocline (Spiegel & Zahn 1992), i.e., the transition zone between the differentially rotating convection zone and the nearly uniformly rotating radiative interior. To estimate the
true width of the transition, the finite resolution of the inversion, as characterized by the averaging kernels, must be taken into account. Kosovichev (1996a) carried out a fit to early data on rotational splittings, characterizing the transition with the function

$$\Phi(r) = 0.5 \left[ 1 + \text{erf} \left( \frac{2(r - r_0)}{w} \right) \right],$$

where $r_0$ is the central part of the transition, and the width $w$ is the characteristic thickness corresponding to a change of $\Phi$ from 0.08 to 0.92. He obtained a width of $(0.09 \pm 0.05) R$ and a central position of $r_0 = (0.692 \pm 0.005) R$, just below the base of the convection zone, which has been localized to $(0.713 \pm 0.003) R$ by helioseismic analyses (e.g., Christensen-Dalsgaard, Gough & Thompson 1991). Analyses of more recent data (e.g., Antia, Basu & Chitre 1998; Basu 1997; Corbard et al. 1998, 1999) have typically yielded a substantially smaller width while confirming that the transition is centered just below the convection zone. For example, Charbonneau et al. (1999), using genetic forward modeling as well as resolution-corrected inversion, obtained $w = (0.039 \pm 0.013) R$ and a position $r_0 = (0.693 \pm 0.002) R$ at the equator. They also found that the tachocline is prolate, with the high-latitude transition taking place at a slightly larger $r_0$; Antia, Basu & Chitre (1998) found indications of a similar, although barely statistically significant, variation in $r_0$.

### 3.3. Radiative Interior

Inferences of the rotation of the deep solar interior depend on the availability of reliable low-degree splittings. These are difficult to obtain due to the small number of $m$ values available and the resulting small range, often comparable to the natural width of the solar oscillation peaks, over which the splitting is determined (Appourchaux et al. 2000, Chaplin et al. 2001). Furthermore, as a result of the weak sensitivity of the splittings to the core rotation (but see Figure 5), even small changes in the splittings have large effects on the inferred rotation rates. Thus, it is hardly surprising that, as reviewed by Eff-Darwich & Korzennik (1998), a rather wide range of results on the rotation of the deep interior of the Sun has been obtained (see also Charbonneau et al. 1998; Corbard et al. 1997; Eff-Darwich, Korzennik & Jimenez-Reyes 2002; Tomczyk, Schou & Thompson 1995). However, recent observations tend to indicate that the radiative interior rotates essentially uniformly. As an example, the left part of Figure 7 shows results obtained by Chaplin et al. (1999) from analysis of a combination of BiSON and LOWL data. Chaplin et al. (1999) used a special version of the OLA technique to localize, as far as possible, the averaging kernels to the core. Although the localization was fairly successful, the consequence was that the error in the inference became substantial (as is evident in Figure 7). The results are clearly consistent with constant rotation of the deep radiative interior at a rate somewhat lower than the equatorial surface rate; however, one might be tempted to infer, as did Elsworth et al. (1995), tentative evidence for a slowly rotating solar core.
Although rotation of the core is uncertain, the available results are sufficient to constrain quite tightly the rotational distortion of the solar gravitational potential, as measured by the $J_2$ coefficient. Recent analyses (Pijpers 1998, Roxburgh 2001) showed that the distortion of the Sun’s external gravitational field has negligible influence on tests, based on Mercury’s orbit, of Einstein’s theory of general relativity (see also Nobili & Will 1986).

Helioseismic studies of the Sun’s rotation have generally implicitly assumed that the Sun rotates on a single axis. Sturrock & Bai (1991) have suggested that the radiative interior may rotate on an axis that is severely inclined to the surface’s rotation axis. The helioseismic data, however, give no support to this suggestion: in fact, they constrain any such oblique rotator to a small fraction (the inner 0.2 $R_\odot$) of the solar interior (Goode & Thompson 1992, Gough & Kosovichev 1993, Gough, Kosovichev & Toutain 1995).

### 3.4. Variations with Time in Solar Rotation

Closer inspection of the variation of the rotation rate with latitude shows departures from a smooth behavior. Kosovichev & Schou (1997) found regions of slightly faster and slower rotation, which they called zonal shear flows. Birch & Kosovichev (1998) and Schou et al. (1998) further analyzed these flows. With the availability of more extended data from the MDI instrument, Schou (1999) investigated the variation with time of the flows by analyzing 72-day segments of observations. He showed that the flows migrated toward the solar equator in a manner very similar to the surface torsional oscillations found by Howard & LaBonte (1980). Furthermore, he found stronger flows, moving toward the pole, at high latitudes. Antia & Basu (2000), Howe et al. (2000a), and Vorontsov et al. (2002) have carried out more detailed analyses, showing that the flows are coherent over at least the outer one third of the convection zone.

As an example, Figure 8 shows the evolving flows as obtained from analysis by Vorontsov et al. (2002) of data from the MDI instrument. The evolution with time was inferred by inverting the data in short (72-day) chunks after subtracting 360-day splittings data for 1996. These analyses indicate that at low- and mid-latitudes, bands of slightly faster rotation migrate toward the equator over the solar cycle, whereas at high latitude there is a strengthening region of faster rotation: most of the convection zone, even at high latitudes and at great depths, seems to participate in the temporal variation. (We recall that the inversion senses only the component of rotation symmetric around the equator. For clarity, both hemispheres of such symmetrized results are illustrated in Figure 8.) Fitting the temporal variation with a sinusoid with an 11-year period indicates that it is consistent with such a period. Howe, Komm & Hill (2000) found evidence for similar flows in earlier helioseismic data. Furthermore, they demonstrated that the helioseismic results were consistent with direct surface measurements of the torsional oscillations. Interestingly, the location of the low- and mid-latitude flows also seems to coincide with the location of the emergence of sunspots, similarly shifting toward the equator as...
the solar cycle progresses in the so-called butterfly diagram. However, the possible physical relation between the flows and the emerging active regions is as yet unclear.

Further analysis, considering again residuals in the rotation rate inferred for individual time segments from the mean over these segments, found evidence for time variations near and below the base of the convection zone, with shorter periods (Howe et al. 2000b). Recent results on these variations are illustrated in Figure 9. Data sets from the GONG and the MDI instruments, both analyzed with RLS and SOLA techniques, show roughly periodic variations with a period of 1.3 years in the equatorial region at \( r = 0.72 \) \( R \) and, more weakly and with the opposite phase, at \( r = 0.63 \) \( R \). Somewhat more erratic variations, with slightly shorter periods, are seen at higher latitudes. Further analyses (e.g., Toomre et al. 2003) have indicated that the variations are not artifacts of the inversion procedures and that the periods are significantly different from 1 year. Also, as is evident from Figure 9, the amplitude of the variation, at least at the equator, appears very substantially reduced from early 2001. However, although Basu & Antia (2001) found superficially similar variations in a somewhat different analysis of the same data, they argued that the variations were not statistically significant.

4. ROTATION AND FLOWS FROM LOCAL HELIOSEISMIC ANALYSIS

The results presented so far, based on global helioseismology, suffer from several limitations. Determination of the oscillation frequencies requires the analysis of a fairly long time series of data to separate in frequency the individual modes and to achieve sufficiently high frequency precision. The resulting frequencies must be time averages over the possibly changing solar structure and dynamics, evidently restricting the time resolution that can be obtained in analyses such as those reported in the previous section. Furthermore, the global-mode frequency splittings constitute a longitudinal average over the Sun, and are sensitive only to that component of the rotation that is symmetric about the equator.

The solar rotation is not precisely north-south symmetric, and there are flows associated with, for example, large convective eddies, which are not axisymmetric and which vary on timescales that are short compared with a rotation period. These can be investigated through local helioseismology. Rather than being decomposed in global modes, of the form given in Equation 2, the observed surface variations can be analyzed in terms of local wave fields that reflect the properties of the solar interior over restricted regions (Gough & Toomre 1983). In one technique for utilizing these properties of the wave (the ring-diagram procedure), a wavenumber-frequency analysis is carried out over limited regions of the solar surface. The observed relation between wavenumber and frequency is assumed to reflect a horizontal average of the structure and flow underlying the given region and can be inverted to determine the radial variation of that structure and flow.
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(e.g., Hill 1988). By carrying out the analysis for many such regions, the variation of the subsurface properties with position on the solar disk can be investigated. In the time-distance technique, initially developed by Duvall et al. (1993), the travel time of wave packets between different points on the solar surface is determined through a correlation analysis. In a manner similar to geoseismic tomography, the set of travel times may then be analyzed to infer the three-dimensional structure and flow field beneath the solar surface (for reviews, see Kosovichev, Duvall & Scherrer 2000, Kosovichev & Duvall 1997). A closely related technique is known as helioseismic holography, or acoustic imaging. It was initially proposed by Roddier (1975) and first applied by Chang et al. (1997) and Lindsey & Braun (1997). (For a review of helioseismic holography, see Lindsey & Braun 2000a.)

4.1. Evolving Zonal and Meridional Flows

Longitudinal averages of the flow field determined from local helioseismology provide a measure of solar rotation, without the imposition of north-south symmetry and indeed show departures from such symmetry. As an example, Figure 10a shows zonal flows inferred from ring-diagram analysis (Haber et al. 2002). A symmetrized version of these results would be quite similar to the results of global helioseismology; in particular, one might note the convergence toward the equator

Figure 10  (a) Variation of the mean zonal flows of solar-surface weather (SSW) with latitude at depths of 1 Mm (dashed) and 7 Mm (solid) as sampled in several-month intervals in the years 1997, 1999, and 2001. The zonal flow is measured relative to the surface-rotation rate and for clarity is offset by a constant that varies with depth. (b) The meridional flow as a function of latitude and depth, with gray regions representing southward flow and white regions indicating northward flow. The mean meridional flow is consistently poleward (southward) in the southern hemisphere (negative latitudes). It is likewise poleward near the surface in the northern hemisphere, but at greater depths in the later years the circulations are reversed within a submerged mid-latitude cell. [Adapted from Haber et al. (2002).]
of the flows between 1997 and 2001. However, there are clearly very substantial departures from symmetry.

Subsurface meridional flow, from the equator toward the poles, was detected with time-distance helioseismology by Giles et al. (1997) and with ring-diagram analysis by González Hernández et al. (1998). Duvall & Kosovichev (2001) investigated the meridional flow in essentially all the convection zone; even at the greatest depths considered, no return flow was found. The meridional flow appears to show substantial variations during the solar cycle (Chou & Dai 2001, Haber et al. 2002). An example of these results is shown in Figure 10. Interestingly, Beck, Gizon & Duvall (2002) found a time-varying component of the meridional flow that was strongly correlated with the torsional oscillations (i.e., the zonal flows) similarly converging toward the equator with the passage of time.

4.2. Solar Subsurface Weather

Local helioseismology has also allowed investigations of more complex flows below the solar surface. Using time-distance analysis, Kosovichev (1996b) and Duvall & Gizon (2000) investigated the properties of supergranular flows, with a scale of approximately 30,000 km. Flows on larger scales, apparently closely related to magnetic active regions, have been found with the ring-diagram technique. This led to the discovery of large-scale structured and evolving flows of solar subsurface weather (SSW) in the near-surface shear layer, as illustrated in Figure 11. SSW involves intricate flow patterns that can change daily, accompanied by more gradually evolving features such as banded zonal flows and meridional cells with reversing circulations. The complex evolution and meandering of these flows may be associated with the largest scales of deep convection (see the results of numerical simulations, in Section 6.4.1). The flows appear to interact with the magnetic fields visible at the surface, with active regions appearing as zones of convergent flow and possible subduction. This suggests that the subsurface flows have the potential to mechanically twist and displace field lines, possibly leading to unstable magnetic configurations that may flare or erupt as coronal mass ejections. Such flows may also have a role in the systematic transport of angular momentum near the top of the convection zone, which can thus be assessed for the first time.

5. PHYSICAL PRINCIPLES

The helioseismic results illustrated in Figures 3 and 7 reveal a substantial differential rotation in the convective envelope of the Sun but almost uniform rotation in the radiative interior. The physical processes responsible for establishing the rotation profile in these two regimes must therefore be quite different. The implication is that convection must redistribute angular momentum in the envelope, and some other processes (see Section 7, below) must drive the radiative interior toward uniform rotation. The tachocline would then be an internal boundary layer between these two distinct regimes.
The subsurface shear layer represents another rotation regime that is distinct from the deep interior, the tachocline, and the bulk of the convection zone. The coupling between these regimes determines the global rotation profile in the solar interior. Gilman, Morrow & DeLuca (1989; see also Gilman 2000a) proposed a possible scenario in which the latitudinal differential rotation in the envelope is maintained by equatorward angular-momentum transport in the bulk of the convection zone and reduced in the tachocline and subsurface shear layer where angular momentum is transported back toward the poles. If such angular-momentum cycles are indeed present, they must be in a dynamical equilibrium to account for the apparent lack of any substantial long-term variation in the solar differential rotation on timescales of decades to centuries. This suggests that the dynamical timescales operating in each rotation regime may be comparable.

5.1. Redistribution of Angular Momentum

If the convective motions in the solar envelope were to conserve their angular momentum, we would expect the angular velocity to increase as the rotation axis is approached. This is not the case. Instead, the angular velocity monotonically decreases with latitude, with relatively little radial variation throughout the convection zone. To gain some insight into why this occurs, we must look to the equation that expresses conservation of momentum in a magnetized fluid:

\[ \frac{\partial \mathbf{v}}{\partial t} = -\rho (\mathbf{v} \cdot \nabla) \mathbf{v} - \rho \nabla p + \rho \nabla \Phi - 2\rho \Omega_0 \times \mathbf{v} + \frac{1}{4\pi} \mathbf{j} \times \mathbf{b} + \mathbf{D}, \]  

(9)
in a frame rotating with steady angular velocity \( \Omega_0 \), where \( \Omega_0 \) is the mean solar rotation (averaged over space and time), \( \mathbf{v} \) is flow velocity, \( \rho \) is density, \( p \) is pressure, \( \Phi \) is the gravitational potential including a centrifugal component, \( \mathbf{j} \) is current density, \( \mathbf{b} \) is magnetic field, and \( \mathbf{D} \) denotes viscous dissipation. The spherical coordinates include radius \( r \) (positive outward), colatitude \( \theta \) (positive southward), and longitude \( \phi \) (positive eastward, in the direction of rotation). In the inertial frame, the angular-velocity profile is given by

\[ \Omega(r, \theta, t) = \Omega_0 + \langle \frac{v_\phi}{r \sin \theta} \rangle, \]  

(10)
where the angular brackets denote an average over longitude. We can also define the angular-momentum density as

\[ J = \rho r^2 \sin^2 \theta \Omega. \]  

(11)

An equation expressing the conservation of angular momentum can be derived by multiplying the zonal component of Equation 9 by the moment arm \( r \sin \theta \) and integrating over longitude (using also the equation of mass conservation). The result can be written as follows:

\[ \frac{\partial J}{\partial t} = -\nabla \cdot (\mathbf{F}_{MC} + \mathbf{F}_{RS} + \mathbf{F}_{MS} + \mathbf{F}_{VD}). \]  

(12)
where the angular-momentum flux includes contributions from the axisymmetric meridional circulation

\[
\mathbf{F}_{\text{MC}} = J (\langle v_{\theta} \rangle \hat{\theta} + \langle v_r \rangle \hat{r}),
\]

(13)

Reynolds stresses

\[
\mathbf{F}_{\text{RS}} = r \sin \theta (\langle \rho v_\theta' v_\phi' \rangle \hat{\theta} + \langle \rho v_r' v_\phi' \rangle \hat{r}),
\]

(14)

large-scale magnetic torques and Maxwell stresses

\[
\mathbf{F}_{\text{MS}} = \frac{r \sin \theta}{4\pi} (\langle b_\theta b_\phi \rangle \hat{\theta} + \langle b_r b_\phi \rangle),
\]

(15)

and viscous diffusion

\[
\mathbf{F}_{\text{VD}} = -\nu \rho r^2 \sin^2 \theta \nabla \Omega.
\]

(16)

In deriving Equations 12–16, we have neglected longitudinal variations of the density \( \rho \) and kinematic viscosity \( \nu \) so they can be factored out of the longitudinal averages. This is an excellent approximation in the solar interior where thermodynamic variations induced by nonaxisymmetric flow fields are expected to be at least four orders of magnitude smaller than the background stratification. Reynolds stresses are often associated with turbulence, but they can also arise from wave-induced momentum transport.

Simple theoretical considerations suggest that the molecular viscosity of the solar plasma is on the order of 1 \( \text{cm}^2/\text{s} \) (e.g., Kippenhahn & Weigert 1990), which implies viscous fluxes that are many orders of magnitude smaller than convective momentum fluxes. Estimates of the Reynolds number in the solar convection zone are typically at least \( 10^{12} \). Consequently, convection in the Sun is thought to be essentially inviscid and therefore highly turbulent on the scales that dominate the energy content and the momentum and heat transport.

Whereas the viscous term in Equation 9 is likely negligible throughout the solar interior, the importance of the magnetic term probably varies greatly. Lorentz forces may play a primary role in maintaining the rotation profile in the deep interior and tachocline. Magnetic stresses and Alfvén waves could also affect the rotation profile in the upper convection zone by transferring angular momentum to the corona and solar wind in open-field regions and among different latitudes in closed-field regions. However, apart from isolated regions of concentrated flux, the magnetic energy density in the convective envelope is much less than the kinetic energy density, so it is currently uncertain whether these processes can compete with convective momentum transport to alter the mean rotation profile significantly and systematically.

If magnetic fields do indeed play a key role in establishing the solar differential rotation, one might expect substantial systematic variations in the rotation profile over the course of the magnetic activity cycle. Such variations are not observed, except possibly in the tachocline and radiative interior. The steady nature of the
solar internal rotation in the convective envelope may be difficult to explain with a magnetically dominated model.

If we neglect the viscous and magnetic terms in Equation 9 and assume that the flow is in a statistically steady state, then angular-momentum transport by Reynolds stresses must offset advection of the mean rotation by meridional circulation. Thus, given the Reynolds stresses and meridional circulation in a particular flow, one can use Equation 12 to compute the equilibrium rotation profile. Conversely, given the rotation profile and the meridional circulation, one can compute the Reynolds stresses necessary to maintain that profile. As a result, the Reynolds-stress divergence that must be operating in the solar convection zone can be estimated using helioseismic determinations of the angular-momentum profile $J$ and plausible meridional circulation patterns that are consistent with current helioseismic and surface observations.

Unfortunately for practical implementation of the above strategy, however, the circulation in the deep interior is poorly known. Also, there is both observational evidence (Haber et al. 2002) and indications from numerical experiments (Elliott, Miesch & Toomre 2000; Miesch et al. 2000) that the circulation may fluctuate substantially with time.

5.2. Baroclinicity and Thermal Wind

An important constraint on the differential rotation profile that can be achieved in a rotating fluid applies if the rotation is rapid enough for Coriolis accelerations to dominate over nonlinear advection. This limit can be expressed in terms of the Rossby number $R_o$:

$$R_o \equiv \frac{\omega_{\text{rms}}}{2\Omega_0} \ll 1,$$

(17)

where $\omega_{\text{rms}}$ is the root-mean-square vorticity of the flow in the rotating frame. If we also assume that viscous dissipation and Lorentz forces are negligible and that the flow is in a statistically steady state, then the zonal component of the curl of Equation 9 becomes

$$\Omega_0 \cdot \nabla \Omega = \left[ \frac{\nabla p \times \nabla \rho}{\rho^2 r \sin \theta} \right].$$

(18)

If pressure and density isosurfaces coincide, the right side vanishes and Equation 18 reduces to the Taylor-Proudman theorem, which implies that the angular velocity must be constant on cylinders aligned with the rotation axis (Pedlosky 1987). This would be the case, for example, in a strictly isentropic (adiabatic) convection zone where pressure is given as a function of density.

In general, however, the surfaces of constant pressure are inclined relative to the surfaces of constant density, giving rise to a nonzero baroclinic term on the right side of Equation 18. The presence of baroclinicity then drives a transient meridional circulation that will redistribute angular momentum. Baroclinicity can be related to latitudinal variations of temperature or entropy if assumptions are made about
the equation of state and the mean stratification (Durney 1999, Kitchatinov & Rüdiger 1995, Weiss 1965). For example, under the assumption of an ideal gas with a constant specific heat and a hydrostatic background stratification, Equation 18 becomes

\[ \Omega_0 \cdot \nabla \Omega = \frac{g}{2 C_p r^2 \sin \theta} \langle \frac{\partial S}{\partial \theta} \rangle. \]  

(19)

This is the equation of thermal-wind balance, which states that zonal flows are maintained by latitudinal entropy gradients, as is often realized to some extent in planetary atmospheres (Pedlosky 1987). The essential assumption here is that nonlinear advective processes (Reynolds stresses) are ignorable. If latitudinal entropy gradients are weak, Equation 19 implies cylindrical rotation profiles.

The solar angular-velocity profiles inferred from helioseismology are clearly not cylindrical (see Figure 3). Some departures from cylindrical alignment are probably due to thermal-wind effects (see Section 6.4.3, below). The stratification in the convection zone is nearly adiabatic, but latitudinal entropy gradients could be established owing to the influence of rotation on the convective heat flux. If Coriolis forces tend to redirect motions perpendicular to the rotation axis, the convective efficiency could be inhibited at low latitudes where the rotation vector is nearly horizontal. This would produce a relatively cool equator and warm poles, which (in Equation 19) could drive a solar-like differential rotation with a monotonic decrease in the angular velocity with latitude. Centrifugal and other effects may also contribute to baroclinicity, but they are likely to be minor relative to anisotropies induced in the convective heat flux by rotation.

Is the solar differential rotation a consequence of thermal-wind balance? Probably not entirely. The turbulent character of solar convection implies the existence of vorticity over a range of length and timescales. The rotation-dominated condition (Equation 17) and therefore the Taylor-Proudman balance (Equation 18) must break down for many flow structures, particularly near the top of the convection zone and in localized downflow lanes and plumes. Reynolds stresses from such flow structures could produce noncylindrical rotation profiles independently of thermal-wind effects.

Helioseismic measurements currently provide little insight into the relative roles of Reynolds stresses and baroclinicity in maintaining the solar differential rotation. If the rotation profile is indeed in thermal-wind balance, Equation 19 implies equator-to-pole temperature variations of approximately 5 K near the base of the convection zone and between approximately 1 K and 2 K near the top. This in turn implies sound speed variations of roughly one part in 10^6 near the base of the convection zone: these are beyond the current sensitivity of helioseismology. The corresponding variations near the top of the convection zone are approximately one part in 10^4, which may be detectable (Kuhn & Libbrecht 1991). However, the near surface layers are less likely to be in thermal-wind balance because the convective motions are more rapid and their Rossby numbers may be large.
6. CONVECTIVE ENVELOPE

The highly turbulent nature of solar convection poses a formidable challenge to theorists and modelers of the dynamics of the convective envelope. Anisotropies induced by rotation and stratification further complicate the dynamics. In the face of such complexity, numerical simulations must play an essential role as in other areas of turbulence research.

However, molecular dissipation scales on the Sun are at least six orders of magnitude smaller than the depth of the convective envelope. A direct numerical simulation of solar convection would have to resolve this entire range of scales in each of three dimensions, a monumental task that is beyond the capability of current or foreseeable computing resources. Consequently, all solar differential rotation models must involve some approximations regarding momentum and heat transport by turbulent motions that are not explicitly computed. Thus, modeling approaches can be classified as either mean-field models or large-eddy simulations (LES) according to the nature and scope of these approximations.

6.1. Mean-Field Models

Mean-field or Reynolds-averaged approaches begin by separating the state variables into mean and fluctuating components. It is then generally assumed that the characteristic length and timescales of the fluctuating flow component are much smaller than those of the mean-flow component (scale separation). Evolution equations for the mean flows are then derived by averaging the equations of motion over space and, often, time. These Reynolds-averaged equations for the mean flows involve second-order correlations among the velocity and thermodynamic fluctuations that take the form of the Reynolds stress \( \langle \rho v_i' v_j' \rangle \) and the convective heat flux \( C_p \langle \rho v' T' \rangle \) (primes denote the fluctuating flow component).

The next step in the mean-field approach is to introduce parameterizations for the Reynolds stress and convective heat flux. A variety of approaches distinguished by their motivation and sophistication have appeared in the literature. All rely on a series of simplifying assumptions about the nature of the flow to arrive at a tractable set of governing equations. Although these assumptions are often questionable, they are generally plausible and can lead to important insight into the nature of rotating, stratified flows. Mean-field models of solar convection have a long history, which has been comprehensively reviewed elsewhere (Canuto & Christensen-Dalsgaard 1998, Rüdiger 1989). Here we review the principal approaches and discuss recent developments.

6.1.1. TURBULENT DIFFUSION AND REYNOLDS STRESSES  Most mean-field formulations incorporate the idea of turbulent diffusion. Reynolds stresses and convective heat flux are typically expressed in terms of a turbulent viscosity and heat conductivity that are in general anisotropic owing to the rotation and stratification. Although viscous diffusion tends to suppress angular velocity gradients
(see Equation 16), it can induce a meridional circulation if the viscosity is anisotropic. This circulation can in turn drive a differential rotation. Likewise, anisotropic heat diffusion can drive a thermal-wind differential rotation. In Cartesian geometries, anisotropic turbulent diffusion is assumed to take the form

\[ \langle \rho v'_i v'_j \rangle = -N_{ijk} \frac{\partial \bar{v}_k}{\partial x_l} \langle \rho v'_l T' \rangle = -\chi_{ij} \frac{\partial \Delta T}{\partial x_j}, \]

where overbars denote mean fields, and \( \partial \Delta T / \partial x_j \) denotes the superadiabatic gradient of the mean temperature (Kitchatinov, Pipin & Rüdiger 1994). These expressions involving fourth- and second-rank tensors possess a large number of parameters, but in practice, they are drastically simplified using symmetry and other arguments.

Mean-field models typically also include nondiffusive components of the Reynolds stresses, which can drive a differential rotation more directly. The most familiar example of this is the \( \Lambda \)-effect, which accounts for systematic, nondiffusive velocity correlations induced by rotation. In its simplest form, the \( \Lambda \)-effect is proportional to the rotation rate \( \langle \rho v'_i v'_j \rangle = \ldots + \Lambda_{ijk} \Omega_k \) (Rüdiger 1989). Higher powers of \( \Omega \) may also appear (e.g., Tuominen & Rüdiger 1989). Other nondiffusive components of the Reynolds stress that have been considered take into account systematic velocity correlations induced by stratification and shear (Canuto, Minotti & Schilling 1994, von Rekowski & Rüdiger 1998).

The concept of turbulent diffusion often has its roots in mixing-length theory, which conventionally assumes that turbulent heat and momentum transport are achieved by eddies of a characteristic size, known as the mixing length. Most mixing-length theory formulations used in stellar structure models apply only to the spherically symmetric convective heat flux, so they are of little relevance to the problem of differential rotation. (For the convective heat flux to produce a thermal wind, it must depend on latitude.) However, Durney (1991, 1996) has developed a version of mixing-length theory that takes into account the distortion of eddies by rotation and shear. The resulting Reynolds stresses can be interpreted in terms of an anisotropic viscosity and a \( \Lambda \)-effect.

Kitchatinov & Rüdiger (1993) derived an expression for the \( \Lambda \)-effect in rotating, stratified fluids using the linearized equations for the fluctuating flow component and a phenomenological turbulence model based on the ideas of scale separation and quasi isotropy. Using similar quasi-linear, quasi-isotropic, mixing-length, and scale-separation arguments, Kitchatinov, Pipin & Rüdiger (1994) then derived the anisotropic turbulent viscosity and heat conductivity tensors for rotating, magnetized fluids.

More sophisticated models may include additional evolution equations for the second-order correlations or moments. These involve third-order moments that are in turn parameterized or otherwise modelled. Canuto, Minotti & Schilling (1994) have developed such a model that includes evolution equations for the turbulent kinetic energy, temperature variance, and energy dissipation rate. The system may be solved by assuming that third-order moments are diffusive to lowest order, with
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Figure 12  (a) Reynolds stresses and (b) latitudinal heat conductivity derived from the mean-field model of Canuto, Minotti & Schilling (1994). The sign of these correlations implies angular-momentum transport that is inward and equatorward and convective heat transport that is poleward (the convective heat transport also includes an outward component that is not shown). [Adapted from Canuto, Minotti & Schilling (1994).]

nondiffusive corrections derived from the governing dynamical equations through an iterative procedure. Canuto, Minotti & Schilling (1994) reported results from several simplified versions of this more complete system by using algebraic parameterizations of the Reynolds stress and convective heat flux. The Reynolds stresses from one such model are illustrated in Figure 12a. To drive a solar-like differential rotation, the Reynolds stresses must transport angular momentum toward the equator, which implies a positive $\langle \rho v_\theta' v_\phi' \rangle$ correlation in the northern hemisphere. This only occurs when both buoyancy and rotation are incorporated into the Reynolds-stress parameterizations. (If either or both of these is neglected, the transport is poleward.) The radial angular-momentum transport is generally inward, as indicated by the negative $\langle \rho v'_r v'_\phi \rangle$ correlation. These results are qualitatively similar to the Reynolds stresses reported by Durney (1991) in his mixing-length model. The Canuto, Minotti & Schilling (1994) approach also includes an anisotropic heat conductivity that is determined from the Reynolds stresses and the turbulent kinetic energy. The results illustrated in Figure 12b imply a poleward convective heat transport that could induce a thermal wind.

Canuto, Cheng & Howard (2001) have recently developed an improved analytic model for the third-order moments that, unlike previous models, does not rely on the quasi-normal approximation for the fourth-order moments. This new model is simpler in form than previous models and is therefore easier to incorporate into stellar structure codes. Furthermore, it gives better agreement with large-eddy simulations of the earth’s convective boundary layer. Applications to solar convection may produce improved treatments, particularly for the overshoot region (Marik & Petrovay 2002).

6.1.2. MODEL ROTATION PROFILES  When mean-field approaches are used to deduce the resulting global differential rotation, they may or may not produce solar-like profiles with radius and latitude. Furthermore, many mean-field models yield
different results on the relative roles of the anisotropic heat transport, anisotropic turbulent viscosity, and the nondiffusive components of the Reynolds stress. Using a model based purely on turbulent diffusion, Pidatella et al. (1986) found that an anisotropic viscosity was more effective than an anisotropic heat conductivity at driving solar-like rotation profiles. However, the required anisotropy implies a horizontal viscosity that is larger than the vertical, a result that is counter-intuitive for a convective system. Küker, Rüdiger & Kitchatinov (1993) produced solar-like rotation profiles that were maintained primarily by a $\Lambda$-effect. However, these computations did not include meridional circulation, which is the mechanism for driving the system toward Taylor-Proudman balance in the rapid-rotation limit (see Section 5 above). When meridional circulation and anisotropic heat transport were considered, Kitchatinov & Rüdiger (1995) found that the former was the dominant effect and departures of the rotation profile from cylindrical alignment were in thermal-wind balance. In their model, the radial turbulent heat conductivity peaks at the poles owing to the influence of rotation, resulting in relatively warm poles and a thermal wind that is qualitatively similar to the solar differential rotation. Durney (1999) has also argued that the solar differential rotation is in thermal-wind balance maintained by anisotropic heat transport. In contrast, von Rekowski & Rüdiger (1998) chose to add another nondiffusive component to the Reynolds stresses, the anisotropic kinetic alpha (AKA) effect. These calculations, which neglected thermal-wind effects, pointed to the AKA effect as the main agent for departures from cylindrical alignment, thus producing more solar-like rotation profiles.

The above processes are all capable of contributing to the maintenance of the solar differential rotation. Which process dominates is sensitive to the assumptions made in developing the model. Future progress may require a more unified treatment of such relevant physical processes. This may be carried out within the framework of more complex turbulence models, as discussed by Canuto, Minotti & Schilling (1994) and Canuto & Minotti (2001). The dilemma in all mean-field approaches is how to achieve reliable closure of the system of evolution equations at a tractable level, and this difficulty is shared by all turbulence models (Cambon & Scott 1999, Lesieur 1997).

6.2. Large-Eddy Simulations

An alternative approach to mean-field parameterizations is to explicitly solve the three-dimensional, nonlinear equations that express the conservation of mass, momentum, and energy in a stratified fluid. The main difficulty here is the tremendous range between global and molecular dissipation scales that currently precludes any direct numerical simulations. Momentum and heat flux by motions on scales smaller than the grid spacing of the discretized system [sub-grid-scales (SGS)] must be treated through some modeling procedure, much as in mean-field treatments. The main distinction between this approach and mean-field models is that nonlinear interactions involving a wide range of relatively large-scale motions
are computed explicitly from the fundamental dynamical equations. Because the largest scales generally dominate the energy content and transport properties, the global properties of the flow should eventually become independent of resolution as the resolution is increased, provided that the simulation captures enough scales and that the SGS model is valid. This is the philosophy behind the LES approach to turbulence modeling, which is playing an increasingly important role in engineering and geophysical applications.

In the LES approach, a filtering procedure is applied to the equations of motion. Although similar to Reynolds averaging this procedure is more general and does not require the assumption of scale separation (Lesieur & Métais 1996, Mason 1994). The spatial scale of the filter is typically chosen to be comparable to but somewhat larger than the grid spacing. The simplest SGS models assume that the influence of unresolved motions smaller than the filter scale is diffusive in nature, with an effective turbulent viscosity and a heat conductivity that are orders of magnitude larger than the molecular values. Such parameterizations may be either passive or dynamic, with parameter values chosen based on properties of the resolved flow. For example, Smagorinsky (1963) proposed a now commonly-used SGS model in which the turbulent viscosity is assumed to be proportional to the local strain-rate tensor associated with the resolved motions. This model is justified by the phenomenological argument that turbulent mixing will tend to be most efficient in regions of strong shear that are liable to shearing instabilities.

6.3. Local-Domain Convection Modeling

What range of resolved scales is sufficient to capture the essential dynamics of the large-scale flow? Some insight into this question can be gained from turbulence simulations in Cartesian geometries where efficient numerical techniques (e.g., fast Fourier transforms) permit higher resolution and therefore higher Reynolds numbers relative to spherical geometries. The effects of rotation may be included under the f-plane approximation in which the rotation vector is constant throughout the layer but may be tilted at an arbitrary angle relative to the vertical. Brummell, Hurlburt & Toomre (1996, 1998) investigated in detail the nature of the Reynolds stresses and the generation of mean flows in f-plane simulations of turbulent compressible convection. As these researchers increased the Reynolds number in their simulations, thereby making the flows more turbulent, they found a qualitative change in the nature of the Reynolds stresses. In laminar parameter regimes, buoyancy forces drive vertical flows, that are diverted in the zonal (longitudinal) direction by Coriolis forces. This leads to correlations between the vertical and zonal velocity components $\langle u'_r v'_\phi \rangle$, which transport zonal momentum and maintain a vertical shear. In contrast, in more turbulent parameter regimes, such vertical flows are quickly destabilized, reducing the $\langle u'_r v'_\phi \rangle$ correlation. The effects of rotation are instead experienced by the large-scale structures that arise, namely the vortical downflow plumes that are realized in turbulent compressible convection. Coriolis forces acting on these helical structures tend to tilt them away from the vertical
and toward the rotation axis. Thus, vortical downflows tend to align with the rotation axis, producing correlations between the vertical and latitudinal velocity components $\langle u' v'_\theta \rangle$. Such Reynolds stresses alter the meridional flows, and the action of Coriolis forces on them yields changes in the zonal flows. The differential rotation achieved by the turbulent alignment of plumes may be quite different from that in more laminar systems where it is maintained primarily by Coriolis-induced $\langle u'_r v'_\phi \rangle$ correlations in larger-scale convection cells. Indeed, Brummell, Hurlburt & Toomre (1996, 1998) found that the substantial vertical shear present in their laminar simulations is reduced in their more turbulent cases, yielding a zonal velocity profile in the mid-convection zone more reminiscent of helioseismic results.

These f-plane results suggest that the Reynolds numbers, and consequently the resolution, must be sufficiently high to begin to capture the distinctly turbulent dynamics that must be occurring in the solar convection zone. More generally, nonlinear interactions, instabilities, and self-organization processes (e.g., inverse cascades) in turbulent flows may operate over a wide range of scales, requiring high resolution to accurately model such dynamics. To address the nature of global-scale flows such as the solar differential rotation, the correct spherical geometry is also essential.

6.4. Convection in Rotating Spherical Shells

Early simulations of convection in rotating spherical shells (reviewed by Gilman 1986, 2000; Glatzmaier 1987; Miesch 2000) were limited by available computational resources to relatively laminar parameter regimes. The first studies focused on Boussinesq fluids, but compressibility effects were soon incorporated via the anelastic approximation, which allows for density stratification but effectively filters out high-frequency acoustic waves that would otherwise severely limit the time step. The anelastic approach involves a perturbation expansion around a reference state that is usually assumed to be spherically symmetric, hydrostatic, and slowly varying. The resulting equations are linear in the thermodynamic perturbations but nonlinear in velocity variables. The anelastic approximation is valid as long as the background stratification is nearly adiabatic and the convective velocities are subsonic (e.g., Gilman & Glatzmaier 1981). This is an excellent approximation in the bulk of the solar convection zone, but it breaks down in the surface layers ($r > 0.98 \, R$).

The dominant convective structures in these early laminar simulations (Gilman & Miller 1986; Glatzmaier 1985, 1987) are the so-called banana cells, which are elongated in latitude and possess moderately high azimuthal wave numbers $m \sim 10–20$. Coriolis-induced tilts in these convection cells induce $\langle u'_r v'_\phi \rangle$ correlations that transport angular momentum toward the equator, generating differential rotation. The feedback of the rotational shear on the convection distorts the banana cells, inducing further velocity correlations that alter the Reynolds stresses. The resulting rotation profiles possess a decrease in angular velocity with latitude, much
as in the solar envelope, but they generally exhibit more cylindrical alignment than suggested by helioseismic measurements.

In recent years, high-resolution simulations of rotating convection in full spherical shells have achieved more turbulent parameter regimes, utilizing spherical harmonic $Y^m_l$ expansions up to degree $l \sim 680$ (Brun & Toomre 2002, Elliott, Miesch & Toomre 2000, Miesch et al. 2000). The anelastic approximation is still used and one-dimensional solar structure models provide the initial reference state. Solar values are taken for the heat flux, rotation rate, mass, and radius, and a perfect gas is assumed because the upper boundary of the shell lies well below the H- and He-ionization zones. Some of these simulations have included convective penetration below the base of the convective envelope, but they have not yet incorporated the subphotospheric layers where the effects of the very steep entropy gradient would otherwise favor the driving of very small granular and mesogranular scales of convection. Resolving such scales in a global simulation would require a spatial resolution at least 10 times greater than presently available and the inclusion of ionization and radiative transfer effects that are presently neglected. The upper boundary of the computational domain is placed below the photosphere, at $r = 0.96–0.98 \ R$, so the density at the lower boundary is 60 to 130 times greater than at the upper boundary. SGS dissipation is handled under the diffusive approximation, using a depth-dependent turbulent viscosity and heat conductivity.

Results of such calculations are illustrated in Figures 13–14 and include Case E (presented by A.S. Brun, M.S. Miesch, J. Toomre, unpublished paper), which is quite turbulent, as well as Case AB, which is a lower-resolution (higher dissipation, hence less turbulent) progenitor version of Case E (see Brun & Toomre 2002).

6.4.1. COMPLEX EVOLVING PATTERNS OF CONVECTION

The convection in Figure 13a is dominated by intermittent plumes of upflow and stronger downflow, some possessing a distinctive cyclonic swirl. The dominant role of coherent plumes, first revealed in planar geometry, is now becoming apparent as ever higher Reynolds numbers are achieved. Many downflows extend over the full depth of the zone, changing from wavy downflowing sheets to distinct plumes at greater depths. The convective patterns evolve over fairly short timescales compared to the solar rotation period, and are advected and sheared by the strong differential rotation that they drive. This suggests that the largest global scales of convection are unlikely to be recognizable after one full rotation. Such rapid variation would make the detection of convective flows using helioseismic measurements more difficult (see Section 4, above). Deeper within the convective shell, persistent features are more apparent, and the downflow network is less homogeneous.

The range of convective structures present is especially evident in the squared vorticity field in Figure 13b. Near the equator, latitudinally extended downflow lanes produce vast vorticity fronts that drift prograde faster than the local rotation. At higher latitudes, vorticity is concentrated in localized swirling plumes near the interstices of the downflow network. The strongest downflows also appear in the
temperature field (Figure 13c) as cooler spots and lanes. Global-scale variations in
temperature are apparent as well and exhibit latitudinal banding near the surface
with a warm equator, cool midlatitudes, and a warm pole. Deeper in the convec-
tion zone, the poles are still warm, but the equatorial regions are cooler, and the
latitudinal temperature and entropy profiles are more monotonic ($\partial S/\partial \theta < 0$
in the northern hemisphere).

6.4.2. RESULTING DIFFERENTIAL ROTATION  The rotation profile in this simulation
is shown in Figure 14. Like that in the solar envelope, it exhibits a monotonic
decrease of approximately 30% in angular velocity from low to high latitudes
throughout the convection zone. Angular-velocity contours still exhibit substantial
cylindrical alignment, but vertical angular-velocity gradients are smaller than in
previous modeling efforts, closer to helioseismic inversions. Also significant is
the continued decrease of the angular velocity at latitudes beyond the tangent
cylinder (the cylinder aligned with the rotation axis and tangent to the base of the
convection zone). If the convection tends to conserve its angular momentum, the
angular velocity of fluid elements will increase as they approach the rotation axis,
tending to spin up the poles. This occurs in many solar convection simulations
in both laminar and turbulent parameter regimes. However, it does not appear
to be occurring in the Sun, so the results shown in Figure 14 are a promising
improvement.

6.4.3. BAROCLINICITY AND THERMAL WIND  One must then ask what is maintain-
ing the differential rotation, particularly in the more solar-like cases. It is especially
important to determine to what extent the rotation is in thermal-wind balance as
expressed by Equation 19. Because the convective flows evolve substantially on
timescales less than a rotation period, we might expect the low–Rossby number
condition (Equation 17) and therefore the thermal-wind balance (Equation 19) to
break down.

Although the convection structure generally adjusts rapidly to parameter
changes, the differential rotation in more turbulent cases such as Case E adjusts
more gradually because the Reynolds stresses are weak. This long adjustment
timescale (at least 10–20 rotation periods), together with the substantial tem-
poral fluctuations typically exhibited by nonlinear fluxes, makes it challenging
to achieve a steady angular momentum balance in highly turbulent simulations.
Thus, the maintenance of differential rotation is often best illustrated in less tur-
bulent simulations that have achieved a close balance among the flux terms in
Equation 12.

The extent to which the differential rotation satisfies Equation 19 is illustrated
in Figure 15 for the less turbulent Case AB (discussed in detail by Brun & Toomre
2002). In the lower convection zone of this simulation, departures of the differential
rotation from cylindrical alignment are in approximate thermal-wind balance, but
in the upper convection zone, this balance breaks down. Because a uniform heat
flux is applied at the upper and lower boundary in this and other simulations, the
latitudinal entropy variations that drive the thermal-wind rotation component arise self-consistently from the influence of rotation on the convective flows.

6.4.4. ANGULAR-MOMENTUM TRANSPORT Further insight into the maintenance of the differential rotation in simulations can be gained by considering the contribution of each term in the angular momentum (Equation 12). Because magnetic fields are neglected in these simulations, the angular-momentum transport is achieved by meridional circulations, Reynolds stresses, and viscous fluxes. For the flow to be statistically stationary, these fluxes must balance. Radial and latitudinal angular-momentum fluxes are illustrated in Figure 16 for the same simulation shown in Figure 15. Reynolds stresses and meridional circulation both transport angular momentum outward and are offset by inward viscous fluxes. This is in contrast to many mean-field models that produce an inward angular-momentum transport by Reynolds stresses as shown in Figure 12a (see also Canuto, Minotti & Schilling 1994, Durney 1991). Resolving this apparent discrepancy requires achieving more turbulent parameter regimes in the simulations. The outward transport in laminar convection can be attributed to velocity correlations induced by the shearing effects of differential rotation on banana cells (Gilman & Miller 1986). This process will likely become less important as the convection structure becomes more complex. If the dissipation is reduced, the viscous contribution in Figure 16 will also be reduced, leaving only Reynolds stresses and meridional circulation that must balance one another to produce a stationary rotation profile. If the circulation flux remains outward, Reynolds stresses must reverse as the Reynolds number is

Figure 16 (a) Radial and (b) latitudinal profiles of the angular-momentum fluxes in Case AB, delineating the Reynolds stress (R), meridional circulation (M), and viscous (V) contributions. The radial profiles are averaged over latitude and the latitudinal profiles are averaged over radius. All profiles are also averaged over time and longitude. (c) Time-averaged meridional circulation shown as streaklines. [Adapted from Brun & Toomre (2002).]
increased, thus transporting angular momentum inward. This trend has already been observed in recent simulations (Brun & Toomre 2002).

The latitudinal angular-momentum transport by Reynolds stresses is directed toward the equator (Figure 16b), as in mean-field models and as it must be to drive a solar-like differential rotation. The meridional circulation also produces equatorward angular-momentum transport, but its amplitude decreases drastically at high latitudes. A look at the structure of the circulation (Figure 16c) reveals why. The strongest circulation cells are confined to low latitudes in the upper convection zone, outside the tangent cylinder. Such simulations rarely exhibit a persistent, single-celled, equatorially symmetric meridional circulation structure as assumed in many simplified solar dynamics and dynamo models. Instead, the circulation is generally multicelled in radius and latitude and asymmetric, with week-to-week fluctuations comparable to or larger in amplitude than the multiration mean. Doppler observations of the meridional circulation at the surface of the Sun generally indicate a more steady, systematic poleward circulation. However, helioseismic measurements do indicate substantial temporal variation and asymmetry in the circulation deeper in the convection zone, closer to the computational domain of the simulations (see Figure 10). Indeed, the absence of a single, persistent cell extending from equatorial to polar regions contributes to the monotonic decrease in angular velocity at high latitudes in the more solar-like simulations. Such extended circulation cells would tend to spin up the poles because axisymmetric meridional flows tend to conserve their angular momentum. Further observations are necessary to determine if a multicell structure exists in the Sun.

6.5. Future Refinements in Modeling

Although solar convection simulations have progressed considerably in recent years, substantial improvements are needed. Such simulations are still sensitive to SGS dissipation parameters such as the Reynolds number and Prandtl number, and thus the LES ideal of resolution-independent results has not yet been achieved (Brun & Toomre 2002, Miesch et al. 2000). Furthermore, the solar convection zone is bordered above and below by complex boundary layers that are difficult to incorporate into global-scale models, so their role in the overall dynamics remains unclear. The underlying tachocline layer is discussed next. The vigorous granulation that occurs in the solar photospheric layers will remain outside the scope of global models for some time, although local models have been successful in reproducing the observed properties (e.g., Stein & Nordlund 1998). However, recent progress has been made in achieving supergranular-scale motions in global simulations by DeRosa, Gilman & Toomre (2002). Their computational domain was confined to 0.90–0.98 \( R \) to focus on the dynamics occurring in the subsurface shear layer. The differential rotation profiles achieved in these thin shells exhibit a negative radial angular-velocity gradient that is the same sign but of a smaller amplitude than the gradient that would be achieved if the convection were to conserve its angular momentum. Such profiles are qualitatively consistent with helioseismic inversions and earlier studies based on surface measurements (Foukal & Jokipii
1975) and with numerical simulations (Gilman & Foukal 1979). The extension of such simulations to deal with a deep shell of convection is now being pursued in earnest (A.S. Brun, M.S. Miesch, J. Toomre, unpublished paper).

6.6. Temporal Variations in Rotation

Simulated differential rotation profiles such as that illustrated in Figure 14 vary with an amplitude comparable to observed variations in the solar differential rotation. Weekly and monthly fluctuations in angular velocity are typically a few percent of the mean (∼20 nHz) and are largest at high latitudes where the moment arm is small and where longitudinal averages cover a smaller volume (Miesch 2000). Transient jet-like features are often seen, but there is little evidence in the simulations for systematic latitudinally-propagating flows comparable to solar torsional oscillations (see Section 3.4, above).

Because torsional oscillations appear to be correlated with the solar activity cycle, they may arise from dynamo-generated Lorentz forces (Yoshimura 1981). In a solar mean-field dynamo model, Covas, Tavakol & Moss (2001) found torsional oscillations similar to those observed in the outer parts of the convection zone (but see Figure 8). Some of the simulations also showed evidence for period halving and noted that with three such period halvings, the basic 11-year sunspot period would yield 1.3 years, as may have been observed near and below the base of the convection zone (but see Figure 9).

7. TACHOCLINE AND RADIATIVE INTERIOR

Given the general picture of the evolution of stellar rotation involving wind-driven spin-down of the convection zone, the rotation of the radiative interior must be determined by the transport of angular momentum from the interior to the convection zone. These processes likely also affect or even determine the structure of the tachocline, which provides the interface between the largely uniformly rotating radiative interior and the differentially rotating convection zone in the present Sun.

7.1. Anisotropic Turbulence and Shear

According to helioseismic inversions, the tachocline appears to be located largely in the stably stratified interior, although it may overlap with the overshoot region and convective envelope, particularly at high latitudes. This stable stratification has essential implications for its dynamics, which set it apart from the convection zone. Penetrative convection and instabilities of the differential rotation may generate turbulence in the tachocline, but buoyancy forces would inhibit vertical motions, and the strong rotational influence would contribute to make this a quasi-two-dimensional turbulence. The stable stratification, proximity to the overshoot region, and strong differential rotation also make the tachocline an ideal place for generating and storing magnetic flux, so magnetism likely plays a more substantial
dynamical role here than in the overlying envelope (e.g., Ferriz-Mas 1996, Gilman 2000a, Stix 1991, Tobias et al. 2001).

A striking feature of the tachocline is that it is a thin layer (less than 5% of the solar radius—probably less than 2%, depending exactly on how one defines its width). Spiegel & Zahn (1992) attributed this to turbulence beneath the convection zone, which acts as a turbulent viscosity to suppress the differential rotation. If such a turbulent viscosity were absent or isotropic, thermally driven circulations would cause the tachocline to broaden over much of the radiative interior over the lifetime of the Sun (Elliott 1997, Spiegel & Zahn 1992). In the picture by Spiegel & Zahn, the lack of such a spread would be explained by an anisotropic viscosity that was much more effective in the horizontal direction than in the vertical owing to the quasi-two-dimensional nature of the underlying turbulence.

Shear instabilities represent a potential source of such anisotropic turbulence. Although the vertical shear is thought to be stable to nonmagnetic perturbations owing to the strong stratification (Schatzman, Zahn & Morel 2000), the latitudinal shear may be unstable. Charbonneau, Dikpati & Gilman (1999) found that the tachocline differential rotation is marginally stable to linear, two-dimensional perturbations in latitude and longitude (see also Garaud 2001), but subsequent work by these authors indicates that it may be destabilized by three-dimensional effects and magnetic fields. A fully three-dimensional instability calculation has not yet been attempted, but vertical motions can be incorporated in a simplified manner using the shallow-water approximation, which has been adapted to the solar tachocline by Gilman (2000b). Dikpati & Gilman (2001) showed that a tachocline-like latitudinal differential rotation in the shallow-water system is indeed unstable for moderate values of their stratification parameter (reduced gravity). Furthermore, if even a relatively small toroidal magnetic field is present ($\gtrsim 10^3$ G), the tachocline rotation is found to be unstable to both two-dimensional and shallow-water perturbations under a wide variety of field configurations (Dikpati & Gilman 1999, 2001; Gilman & Dikpati 2000; Gilman & Fox 1997, 1999). Although magnetic fields did not play a role in the original Spiegel & Zahn (1992) picture, these instabilities transport angular momentum from the equatorial regions toward the poles primarily via Maxwell stresses and can potentially provide the required anisotropic mixing. Recent two-dimensional simulations by Cally (2001) and Cally, Dikpati & Gilman (2003) indicate that the nonlinear saturation of these MHD shear instabilities is indeed associated with dramatic changes in the field configuration and angular-momentum redistribution that can drive the layer toward uniform rotation in latitude. Cally (2003) has begun to extend these studies to three dimensions and has identified new modes of instability.

The scenario proposed by Spiegel & Zahn (1992) assumes that the turbulence will tend to smooth out the rotation profile, in the manner of a classical viscosity (albeit anisotropically). McIntyre, (1998, 2003) however, pointed out that turbulence in a stratified fluid causes not only short-range momentum transport, which can be characterized by an eddy viscosity, but also long-range momentum transport by waves (so-called radiation stresses), because the turbulence will almost invariably also cause wave motions. By analogy with well-studied examples in the earth’s
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atmosphere, McIntyre (1998, 2003) argued that, in the absence of magnetic fields, the overall effect of such momentum transport is to enhance shear gradients, not to eliminate them [but also see the laboratory experiment by Plumb & McEwan (1978)]. Short-range turbulent mixing that acts as an anisotropic diffusion may still dominate near the overshoot region where advective timescales are shorter than the characteristic periods of internal waves (Miesch 2003), but this region may not be wide enough to have a substantial influence on the global rotation profile.

7.2. Tachocline Confinement by a Remnant Core Magnetic Field

If angular-momentum transport by stratified turbulence in the tachocline and radiative interior is indeed nondiffusive, it cannot account for the nearly uniform rotation profile inferred in the deep solar interior by helioseismology. Gough & McIntyre (1998) instead proposed that the momentum transport uses large-scale circulation to enter the tachocline from the convection zone at low and high latitudes and closes via an upwelling at midlatitudes. This circulation is driven by turbulent stresses in the convection zone and is prevented from penetrating deeper into the solar interior by a magnetic field in the radiative interior of strength

$$B_0 \simeq 5 \times 10^{-15} \left( \frac{r_c}{\Delta} \right)^9 \text{ Gauss},$$

where $r_c$ is the radial location of the tachocline, and $\Delta$ is its width. Assuming $\Delta \simeq 0.018 \, R_\odot$ and $r_c \simeq 0.69 \, R_\odot$ (Elliott & Gough 1999), this implies a field in the outer part of the radiative interior of strength of approximately 1 G. Because the relevant magnetic diffusion timescale is longer than the present age of the Sun, such an internal field could be of primordial origin: a fossil remnant left over from formation (Mestel & Weiss 1987). Kitchatinov & Rüdiger (1996) and Rüdiger & Kitchatinov (1997) have also argued that a remnant magnetic field in the solar interior is necessary to keep the tachocline confined to a thin layer, but their model was based on turbulent diffusion and did not include meridional circulation.

Even though magnetic fields play an essential role in both, there are important distinctions between the approach by Gough & McIntyre (1998) and the shear instability picture devised by Gilman, Fox & Dikpati. According to Gough & McIntyre (1998), the dynamics within the tachocline are essentially nonmagnetic apart from a thin boundary layer that acts as a transition between the static, magnetized interior and a nonmagnetic region that contains the shear and circulation. Furthermore, they argued that angular-momentum transport by the global circulation occurs on extremely long timescales ($\sim 10^6$ years). In contrast, the scenario proposed by Gilman, Fox & Dikpati is intrinsically magnetic throughout and operates on timescales that are more comparable to the rotation period, i.e., months to years. This has led Gilman (2000a) to make the distinction between fast and slow tachocline dynamics.

Mestel & Weiss (1987) have demonstrated that even a weak magnetic field is capable of maintaining uniform rotation in the solar interior, as required by the
Gough & McIntyre model. However, if the internal field extends into the convective envelope, magnetic stresses may transmit some differential rotation to the radiative interior (MacGregor & Charbonneau 1999). Garaud (2002) has argued that the advection of field lines by meridional circulation may help keep them confined to the radiative interior at intermediate field strengths, thus reducing the coupling to the convection zone and allowing a nearly uniform internal rotation profile. In Garaud’s (2002) simplified model, the circulation is artificially driven by Ekman-Hartmann pumping, but thermally induced circulations of the kind proposed by Gough & McIntyre may have a similar effect. Alternatively, Forgacs-Dajka & Petrovay (2002) have argued that an oscillating dipolar field imposed from above the tachocline by dynamo action may potentially keep the differential rotation of the envelope from spreading into the interior.

7.3. Implications for Solar Structure and Evolution

The models proposed to explain the properties of the tachocline all include flows or turbulence in the region just below the convection zone. The mixing that results would affect the composition structure in this region, where the settling of helium, in the absence of mixing, tends to produce a sharp gradient in helium abundance. In fact, such mixing would reduce an otherwise prominent difference between the sound speed in solar models, just below the convection zone, and the solar sound speed as inferred from inversion of multiplet frequencies (e.g., Brun, Turck-Chièze & Zahn 1999, Elliott & Gough 1999). Mixing below the convection zone is also required to explain the depletion of the observed solar-surface abundance of lithium relative to the meteoritic abundance.

The long-term evolution of the rotation of the solar radiative interior has been modelled assuming transport through diffusive processes in which the diffusion results from rotationally induced instabilities (e.g., Chaboyer, Demarque & Pinsonneault 1995; Pinsonneault et al. 1989). At the age of the present Sun, these calculations predicted rotation of the solar interior at a rate several times higher than the surface rate, in stark disagreement with the helioseismic inferences of nearly uniform rotation slightly below the surface equatorial rate. One possible reason for this disagreement is that the angular-momentum transport within the radiative interior is inadequately represented in these models. Alternatively (or in addition), the coupling between the radiative interior and the convective envelope may be missing essential physics. This coupling plays a crucial role in the spin-down of the radiative interior because it can transport angular momentum to the envelope where it is eventually carried away from the star by torques exerted by the magnetized solar wind. Two mechanisms have been proposed to account for the relatively slow rotation of the deep solar interior, as inferred from helioseismology: gravity waves and a primordial magnetic field.

Penetrative convection at the base of the envelope likely generates a spectrum of gravity waves. Those waves with a longitudinal phase speed that approaches the local angular velocity are rapidly damped and may drive oscillating zonal flows analogous to the quasi-biennial-oscillation (QBO) phenomenon in the earth’s
stratosphere (Kim & MacGregor 2001, Kumar, Talon & Zahn 1999). Such dynamics could plausibly account for the observed quasiperiodic oscillations in the angular velocity of the solar tachocline apparent in Figure 9 (although magnetic fields may also play an essential role). If a significant fraction of gravity modes propagate through the tachocline despite the inhibiting effects of shear and magnetic fields, they could transport angular momentum and alter the rotation of the deep interior. Early studies by Kumar & Quataert (1997), Schatzman (1993, 1996), and Zahn, Talon & Mathias (1997) suggested that such gravity-wave transport could account for the relatively slow, uniform rotation of the solar interior. However, as pointed out by Gough & McIntyre (1998) and Ringot (1998), these treatments did not correctly account for the nondiffusive nature of gravity wave transport (see Section 7.1, above). Talon, Kumar & Zahn (2002) developed a revised model that incorporated a QBO–type shear layer near the tachocline driven by the selective dissipation of waves with different phase speeds. They argued that the filtering effects of this shear layer, when combined with turbulent diffusion, could extract angular momentum from the radiative interior and slowly drive it toward uniform rotation on timescales of $\sim 10^7$ years. However, the presence of turbulence in the solar interior and its parameterization by means of a turbulent diffusion remain open to question.

The second class of models invoked to account for angular-momentum transport in the deep solar interior and for its coupling to the convective envelope postulates the existence of a primordial magnetic field. As discussed above (see Section 7.2), such a field will tend to produce a nearly uniform rotation profile if it is largely confined to the radiative interior. In this case, coupling to the convective envelope can occur through a tachocline-like boundary layer owing to Lorentz forces, meridional circulations, and Reynolds stresses that are still typically modelled using a turbulent viscosity. Charbonneau & MacGregor (1993) carried out extensive calculations of the evolution of a magnetized model of the solar interior and found that a broad range of initial conditions would give rise to the observed present solar internal rotation. Rüdiger & Kitchatinov (1996) reported comparable results.

How is one to choose between these spin-down models? Both the gravity-wave and the magnetic scenarios certainly require further theoretical refinement aiming at a more realistic description of the tachocline region. It is likely, furthermore, that stellar observations such as those shown in Figure 1 might provide important insights. In particular, Soderblom, Jones & Fischer (2001) pointed out that the rate of change of surface rotation, as inferred by comparing clusters of different age, may provide a measure of the timescale for coupling between the convection zone and the interior. MacGregor & Charbonneau (1999) argued that such observations favored a relatively weak coupling, as would be produced with a fossil magnetic field confined largely to the radiative interior. Barnes (2003) provided a recent overview of the observational situation. He also proposed a model to account for the distribution of rotation periods and its variation with age in terms of angular-momentum loss in stellar winds driven by different types of dynamo action. Further improvements in such essential observations of stellar rotation can be expected with the new generation of large telescopes with efficient spectrographs.
8. CONCLUDING REMARKS

In his review of solar rotation, Howard (1984) stated the following:

The study of solar oscillations of various sort (‘solar seismology’) promises to provide a great deal of information of the Sun and other stars within the next decade (Fossat 1981). We are on the threshold of a new era in solar and stellar research, with potential results that were undreamed of a decade ago.

We hope the present review has demonstrated that these expectations have been met, as far as the helioseismic results are concerned, to an extent perhaps even surpassing Howard’s expectations. Future helioseismic observations, in particular from NASA’s Solar Dynamics Observatory, will extend local helioseismic studies deeper into the convection zone to enable further advances in understanding the relationship between rotation and solar activity and the assessment of angular-momentum fluxes and Reynolds stresses for comparison with numerical simulations. The coming decade will also see the extension of seismic investigations to other stars, with the launch of several asteroseismology space missions. Meanwhile, the steadily increasing realism of the numerical simulations permitted by advances in high-performance computing technology will undoubtedly advance us toward an understanding of the physical processes controlling the internal rotation of stars. The result will be a new and more complete understanding of stellar evolution and activity in all its aspects.

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Figure 3  Rotation of the solar envelope inferred from observations by MDI using a SOLA inversion technique (Schou et al. 1998). The equator is at the horizontal axis and the pole is at the vertical axis. Contour labels are in nHz. The dashed circle indicates the base of the convection zone and the tick marks at the edge of the outer circle are at 15° intervals. The white region near the rotation axis indicates the region in the Sun where no reliable inference can be made with the current data.

Figure 8  The evolution with time of the zonal flows, relative to the epoch of minimum solar activity, inferred from inversions of data from MDI. [Courtesy of S.V. Vorontsov.]
Figure 9 Variations in time over seven years of the residuals $\delta \Omega / 2\pi$ in rotation rate at three latitudes (equator at top; 30 degrees, middle; 60 degrees, bottom) for two depths, with 0.72 R (left) just above the tachocline and 0.63 R (right) in the radiative interior. Two inversion procedures were used for each, with filled symbols indicating RLS inversion, and open symbols OLA inversion. Results obtained from GONG data are shown as blue circles, from MDI data in red triangles. [Adapted from Toomre et al. 2003.]
Figure 11 Changes in global-scale flows with advancing solar cycle. Synoptic maps of horizontal flows deduced from ring-diagram probing at a depth of 10 Mm for Carrington rotations (a) 1923 (from the year 1997) and (b) 1949 (year 1999). The large-scale patterns possess strikingly different character from year to year as magnetic activity intensifies. In particular, active regions appear as zones of convergence and possible subduction. For latitudes higher than 40° in the northern hemisphere, there exists a band or cell of reversed meridional circulation with equatorward flow starting in 1999. In the southern hemisphere meridional flows are consistently poleward. [Adapted from Haber et al. 2002.]

Figure 13 Three views at one instant in time near the top of the spherical shell ($r = 0.98R$) of turbulent convection (Case E) of (a) the radial velocity (downflows are blue, upflows yellow), (b) the vorticity squared (most intense values are white/yellow), and (c) the fluctuating temperature (warmest is white/yellow, cool is blue/green). [Adapted from A.S. Brun, M.S. Miesch & J. Toomre, submitted manuscript.]
Figure 14 Time-averaged angular velocity $\Omega/2\pi$ profiles with radius and latitude obtained from the three-dimensional deep spherical shell convection simulation (Case E, Figure 13), displayed in (a) as contours (with red tones indicating fast rotation and blue-green the slowest rotation), and in (b) as cuts with radius at the indicated latitudes. Such profiles are now making good contact in the bulk of the convection zone with the helioseismic deductions. [Adapted from A.S. Brun, M.S. Miesch & J. Toomre, submitted manuscript.]

Figure 15 Assessment of baroclinicity and thermal-wind balance in a three-dimensional spherical shell simulation of convection (Case AB), showing the time-averaged (a) longitudinal velocity $\langle v_z \rangle$, (b) its derivative along the rotation axis and (c) the component of this derivative that can be attributed to the baroclinic term in the meridional force balance as expressed in Equation 19. [Adapted from Brun & Toomre 2002.]