32 Stars notes 2019/11/20 - Wed - Neutrinos and advanced burning

Discuss day/time for grad exam (undergrad is Wed 11:30-2), maybe Thu?

Discuss order for mesa projects

32.1 Neutrinos continued

We found that the energy loss to neutrinos is

$$\frac{du_\nu}{dt} = 5 \times 10^{18} \frac{ergs}{cm^3 sec} T_9^3 \exp\left(\frac{-11.84}{T_9}\right)$$

this is density independent which is because the product $n_e n_{e^+}$ only depends on T.

Now we can do the analog of the Kelvin-Helmholtz time. The themal energy

$$E_{th} = nkT = 7 \times 10^{19} \frac{ergs}{cm^3} T_9 \rho_4$$

so the cooling time is

$$t_{cool} \simeq \frac{E_{th}}{du_\nu/dt} = 14 sec \frac{\rho_4}{T_9^2} \exp\left(\frac{11.84}{T_9}\right)$$

for some values

T_9	t_{cool}
0.63	160 years
1	0.7 months
2	30 minutes

once ν cooling kicks in, the stellar evolution is dramatically accelerated, even though the external luminosity has basically not changed.

32.2 Nuclear lifetime of late burning

Compare to how we discussed hydrogen burning lifetime:

$$t_{nuc} = \frac{E_{nuc}/particle}{E_{th}/particle} t_{cool}$$

for sun: $t_{nuc} \simeq MS\ life = 5MeV/1keV = 5000 \times t_{cool}$

For massive star core: $E_{nuc} \simeq 1MeV/m_p$ and $E_{th} \sim 100$ keV and so

$$t_{nuc} \simeq (10-30) t_{cool}$$

Nuclear burning stages are not very long-lived. Cooling time is short, and nuclear lifetime is only 10 or so times the cooling time.

32.2.1 Plasmon Decay, another ν production process

can a free photon pair produce? no can't conserve both energy and momentum (can't boost into the rest frame of the photon). But a photon in a plasma has a dispersion relation:

$$\omega^2 = (ck)^2 + \omega_0^2$$

where

$$\omega_0^2 = \frac{4\pi n_e e^2}{m_e} = \text{plasma frequency}$$

which looks like it has a mass. So it can pair produce. (the plasma takes up the momentum slack.) At high electron densities $\hbar\omega_0 \sim kT$ and photons in the plasma become very modified. In this limit

$$E_\gamma^2 = (p_\gamma c)^2 + (\hbar\omega_0)^2$$

The "excess energy" allows the plasmon to decay into 2 ν's.

32.3 Carbon Burning

For $M \geq 8M_\odot$ ^{12}C in the C/O core ignites and burns, however the condition for an equilibrium is that

$$\epsilon_{CC} = \frac{ergs}{gr - sec} = \epsilon_\nu$$

the process is

$$^{12}C +^{12} C \rightarrow^{24} Mg^* + \gamma$$

the Mg is very unstable and decays to: $\alpha +^{20} Ne$ and $p +^{23} Na$ and $n +^{23} Mg$. So the C burning depletes all the ^{12}C and makes ^{20}Ne, ^{24}Mg and ^{23}Na. so the "ashes" are these three plus O.

The reaction rate is dominated by the coulomb barrier. The non-resonant rate is

$$\langle\sigma v\rangle = \frac{7 \times 10^{19} s}{\mu Z_1 Z_2} e^{-\tau} \tau^2$$

and $\mu = A/2$ here

$$\tau = \frac{3E_0}{kT} = \frac{84.2}{T_9^{1/3}} \left(\frac{Z}{6}\right)^{5/3}$$

and so finally

$$\langle\sigma v\rangle = \frac{690}{T_9^{2/3}} \exp\left[\frac{84.2}{T_9^{1/3}} \left(\frac{Z}{6}\right)^{5/3}\right]$$

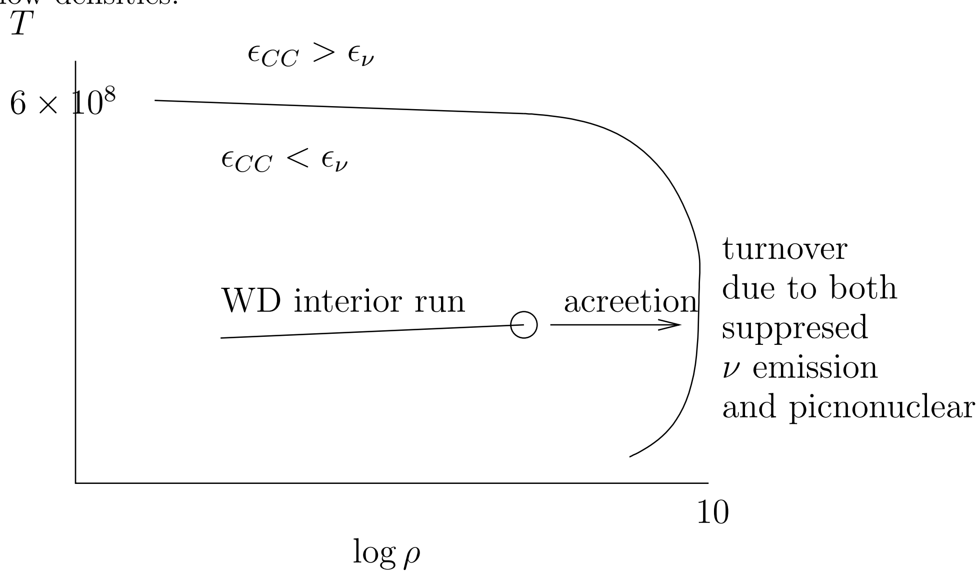
so the energy generation is

$$\epsilon_{CC} = 3 \times 10^{55} \frac{ergs}{gr - sec} \frac{\rho_6}{T_9^{2/3}} \exp\left(\frac{-84.2}{T_9^{1/3}}\right)$$

and for neutrino cooling:

$$\frac{du_\nu}{dt} = 5 \times 10^{18} T_9^3 \exp\left(\frac{-11.84}{T_9}\right) = \epsilon_\nu \rho$$

This is almost entirely temperature determined because that's in the exponent. so then the ^{12}C ignites ($\epsilon_{CC} > \epsilon_\nu$) when $T > 6 \times 10^8$. Figure above: see that carbon ignition is flat at low densities.



at high densities the ignition lines becomes colder.

At high densities the COM energy of 2 ^{12}C nuclei depends on the coulomb physics of the lattice Even at $T = 0$ there is fusion of ^{12}C due to COM energy from zero-point motion in lattice. Reaction becomes very ρ sensative. (this is real cold fusion)

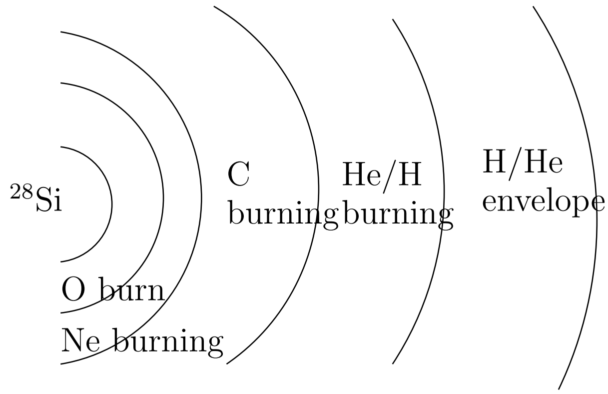
There is a white dwarf drawn on the above plot, and as the center moves to the right due to accretion it eventually reaches the ignition curve. This happens at about the Chandrasekhar mass. These are what Type Ia supernova are thought to be, and this characteristic mass is why they are good standard candles.

32.4 Onion skin structure

So far we have done H, He, C, Ne burning.

See plots in MESA paper 1 for massive star.

These reactions occur in shells in the outer parts of the star.



See Sana et al. 2012, Science, 377, 444, particularly figure 2, showing that most massive star interact with a companion at some point during their evolution.

32.5 Neon "burning" - actually photodisintegration

The star's central T is getting high enough so that

$$\gamma +^{20} Ne \rightarrow \alpha +^{16} O$$

this is called photodisintegration. These alpha particles can be captured on other things for instance

$$\alpha +^{20} Ne \rightarrow \gamma +^{24} Mg$$

and

$$\alpha +^{24} Mg \rightarrow \gamma +^{28} Si$$

so at the end of Ne burning you are pretty much left with these O, Mg and Si. The neon ignition is also on the table with the figure above.

32.6 Oxygen Burning

Oxygen burns next, High coulomb barrier makes it difficult until $T \geq (2-3) \times 10^9$ K.

$$^{16}O +^{16} O \rightarrow^{28} Si + \alpha$$

this also produces ^{31}P+p and ^{31}S+n. So the lightest ash is ^{28}Si.

	$M = 15M_\odot$		$M = 25M_\odot$	
	$L_{ph} = 10^4 L_\odot$		$L_{ph} = 3 \times 10^5 L_\odot$	
Element	L_ν/L_{ph}	time(yr)	L_ν/L_{ph}	time(yr)
C	1	6000	8.3	170
Ne	2000	7	6500	1.2
O	2×10^4	1.7	1.9×10^4	0.5
Si	9×10^5	Week	3.2×10^6	day

These timescales of evolution are longer than the dynamical time

$$t_{dyn} = \frac{1}{\sqrt{G\rho}} = \frac{4 \text{ sec}}{\sqrt{(\rho/10^6)}}$$

So can presume hydrostatic balance. However, the evolution timescales are shorter than the time it takes to reach a thermal equilibrium so one must track thermal evolution.

32.7 ^{28}Si photodisintegration

Until now, we have always released nuclear energy by fusing lighter nuclei together. For pure ^{28}Si, the temperature needed to overcome the coulomb barrier is so high that other physics intervenes.

What happens instead is that you start to chip away at the nuclei with photons.

$$\gamma +^{28} Si \rightarrow^{24} Mg + \alpha$$

Want to get around the coulomb barrier, so the star breaks up some of the Si into α particles and then captures them consecutively onto other Nuclei:

$$\begin{aligned} \alpha +^{28} Si &\rightarrow {}^{32}S + \gamma \\ \alpha +^{32} S &\rightarrow {}^{36}Ar + \gamma \\ &\vdots \\ \alpha +^{52} Fe &\rightarrow {}^{56}Ni + \gamma \end{aligned}$$

in vacuum this ^{56}Ni will then decay to ^{56}Fe. All these reactions are fast, so we will pretty much be able to use the Saha equation.

At what T does a small amount of α lead to an interesting ^{32}S?

$$\mu_{28} + \mu_4 = \mu_{32}$$

these particles are pretty much all ideal.

$$m_{28}c^2 + m_4c^2 - kT \ln \left(\frac{n_{Q,28}}{n_{28}} \frac{n_{Q,4}}{n_4} \right) = m_{32}c^2 - kT \ln \left(\frac{n_{Q,32}}{n_{32}} \right)$$

where

$$Q = (m_{28} + m_4 - m_{32})c^2 = 7MeV$$

and so

$$\frac{n_{28}}{n_{32}} = \frac{4 \times 10^{35}}{n_4} \exp \left[\frac{-16}{(T/5 \times 10^9)} \right]$$

and then

$$\rho_{burn} = 3 \times 10^7 g/cc$$

and for pure He this is $n_4 = 5 \times 10^{30}$/cc. Now set $n_4 = 10^{28}$ i.e. 1% He. this gives

$$\frac{n_{28}}{n_{32}} = \frac{4 \times 10^{35}}{10^{28}} \exp(..) = 5$$

at $T = 5 \times 10^9$. Thus a small amount of αs liberated by photodisintegration can begin the process $^{28}Si \rightarrow ^{56}Ni$. If the burning lasts long enough you can decay via weak processes. Can calculate the equilibrium value of ^{56}Ni, and then use this with the weak decay rate to calculate the production of ^{56}Fe. Since the burning lasts long enough for weak interactions, the end point is ^{56}Fe + some other heavy nuclei.

The evolution after ^{28}Si burning is to an ^{56}Fe core of increasing mass. This is still cooling due to neutrinos, so it contracts and eventually becomes degenerate. Looking at the handout you can see that even when Si burning the core is already degenerate. The pressure is matched by degenerate e^-'s and as M increases the electrons become relativistic. (Recall any object supported by relativistic particles has nearly zero binding energy.) This leads to a rapid collapse once the Fe core mas is $\simeq M_{ch} = 1.4 M_\odot$.

32.8 Chandrasekhar mass (for white dwarfs) and the Fe core

The iron core in high mass stars is again held up by degenerate electrons. In the last stages of stellar evolution, the electrons become degenerate, and eventually relativistic. This limits the mass of the iron core.

Quick white dwarf structure. Fe core is much like a massive Fe WD. Degenerate gas:

$$n_e = \frac{(g=2)}{h^3} \int F d^3p = \frac{8\pi}{3h^3} p_f^3$$

where F is the fermi-dirac distribution function - a step function for fully degenerate electrons. Pressure for non-relativistic:

$$P = \frac{2}{5} n_e E_F = \frac{2}{5} n_e \frac{p_f^2}{2m_e}$$

and with the above

$$P = n_e^{5/3} \left(\frac{3h^3}{8\pi} \right)^{2/3} \frac{1}{5m_e}$$

With $n_e = Z n_i$ and $\rho = (A m_p) n_i$ we get

$$n_e = \frac{Z}{A} \frac{\rho}{m_p}$$

and then call $\mu_e = \frac{A}{Z}$ so that we get

$$P = \left(\frac{\rho}{\mu_e m_p} \right)^{5/3} \left(\frac{3h^3}{8\pi} \right)^{2/3} \frac{1}{5m_e}$$

This would be much the same for a neutron star but μ_e will be one and the electron mass will be replaced by the baryon mass, m_p. Just this gives the ratio of radii between WDs and Neutron stars. We'll talk about this shortly.

Roughly from hydrostatics ($dP/dr = -\rho g$)

$$P_c \simeq \frac{GM}{R^2}\frac{M}{R^2}$$

and $\rho R^3 \simeq M$ so that

$$P_c \sim \frac{GM^2}{M^{4/3}}\rho^{4/3} \simeq \left(\frac{\rho}{\mu_e m_p}\right)^{5/3}\left(\frac{3h^3}{8\pi}\right)^{2/3}\frac{1}{5m_e}$$

so that we find

$$GM^{2/3} = \frac{1}{5m_e}\left(\frac{1}{\mu_e m_p}\right)^{5/3}\left(\frac{3h^3}{8\pi}\right)^{2/3}\rho^{1/3}$$

and then the central density becomes

$$\rho_c \sim G^3M^2\left[\frac{(5m_e)^3(\mu_e m_p)^5}{(3h^3/8\pi)^2}\right] = 4\times10^5\frac{g}{cm^3}\left(\frac{M}{0.1M_\odot}\right)^2\left(\frac{m_x}{m_e}\right)^3\left(\frac{\mu_e}{2}\right)^5$$

where m_x is the mass of the particle supplying the degeneracy pressure. Equating this to $\rho \sim M/R^3$ gives

$$R = 2\times10^9 cm\left(\frac{0.1M_\odot}{M}\right)^{1/3}\left(\frac{m_e}{m_x}\right)$$

this is for $\mu_e = 2$. Now we can say: what if it's neutrons? $m_e/m_x = 1/1800$ so you get 10^6 cm $= 10$ km!

Now we want to look at what happens as the mass increases. As M goes up p_f also increases until eventually the electrons become relativistic. At some point $p_f \geq m_e c$. Can figure out the density at which this occurs,

$$\rho(p_f \simeq m_e c) \sim 10^6 gr/cc$$

Can work in the limit of extremely relativistic $E_f = cp_f$

$$P = \frac{1}{4}n_e E_F = \frac{1}{4}n_e p_f c = n_e^{4/3}\frac{c}{4}\left(\frac{3h^3}{8\pi}\right)^{1/3} \propto \rho^{4/3}$$

scaling changes to $P \propto \rho^{4/3}$.

Now we try to find stuff

$$P \propto \rho^{4/3} \simeq P_c \propto \frac{M^2}{R^4}$$

which gives

$$\frac{M^{4/3}}{R^4} \propto \frac{M^2}{R^4}$$

radius cancels and mass is determined!

$$M_{ch} = 1.456\left(\frac{2}{\mu_e}\right)^2 M_\odot\ .$$

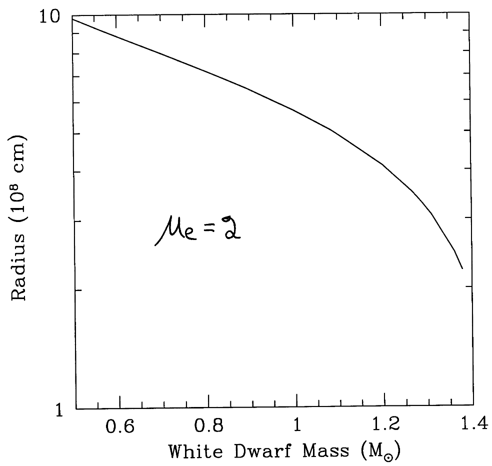


Figure from Hamada and Salpeter has more physics (includes Coulomb corrections) and does this for different compositions. For example for Iron $\mu_e = 52/26$ which is a bit greater than 2 and so the Chandrasekhar mass is $M_{ch} = 1.26 Fe$.

and M_{Ch} denote the radius and mass in the Chandrasekhar approximation for $\mu = A/Z = 2$. The other entries are results using our improved equation of state for He^4 ($\mu = 2.002$), C^{12} ($\mu = 2.001$), and Mg^{24} ($\mu = 1.999$). The entries marked with an asterisk have the highest central densities at which C^{12} or Mg^{24} is stable; later entries refer to models with a core of Ne^{24} and an outer zone of C^{12} and Mg^{24}, respectively. Table 2 gives the results for ${}_{26}Fe^{56}$ ($\mu = 2.152$), together with the Chandrasekhar ap-

TABLE 4*

ZERO-TEMPERATURE MODELS FOR EQUILIBRIUM COMPOSITION

	LOG ρ_c							
	8 627	8 920	9 147	9 361	9 692	10 28	11 28	11 53
(Z; A)	28; 62	28; 64	28; 64	28; 66	28; 66	30; 78	32; 90	38; 120
R.	0 400	0 343	0 300	0 267	0 216	0 157	0 080	0 074
M . ..	1 000	1 011	1 015	1 005	0 990	0 913	0 753	0 711

* (Z; A) denotes the nuclear species at the center (ρ_c in gm/cc, R in units of 0 01$R_\odot$, M in units of $M_\odot$)

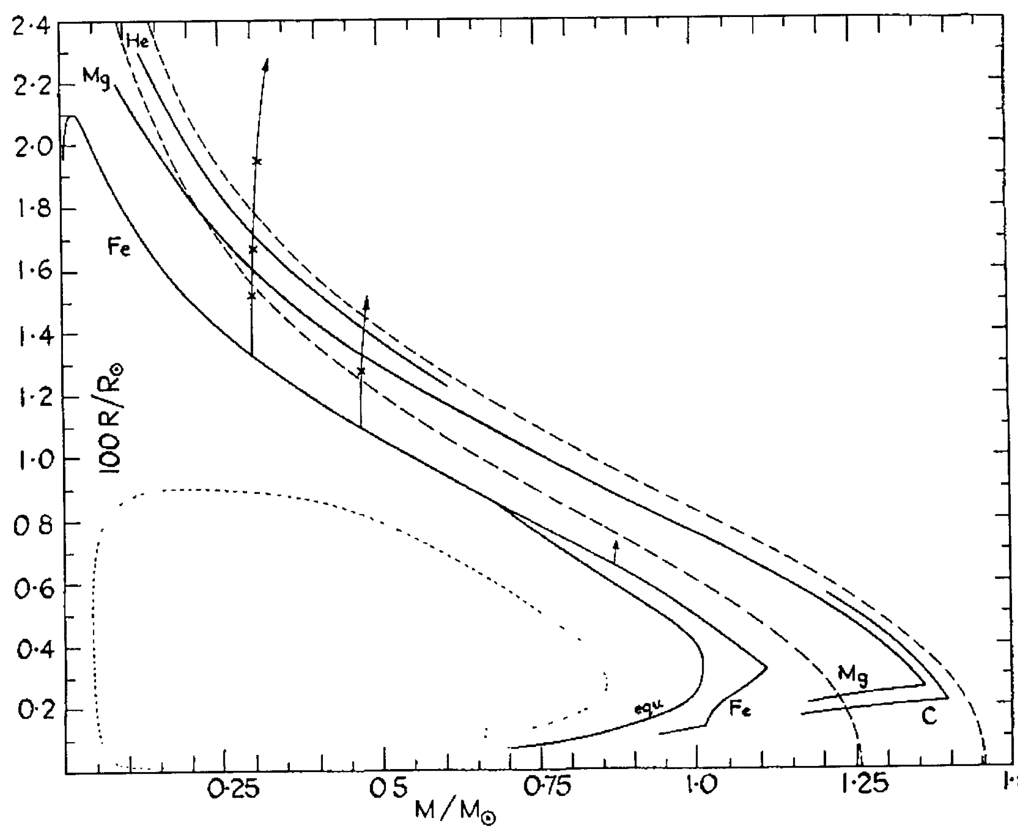


FIG. 1.—The relation between mass M and radius R for zero-temperature stars for He^4, C^{12}, Mg^{24}, and Fe^{56}. The curve marked *equ* denotes equilibrium composition at each density. The dashed curves denote the Chandrasekhar models, the upper one for $\mu = 2$ and the lower one for $\mu = 2.15$. The dotted curves denote neutron stars. The vertical arrows denote stars with H^1 in the outer layers.