28 Stars notes 2019/11/6 - Wed - He burning

28.1 Where we are in evolution

A brief reminder of where we are in evolution:
Helium ignition:
For $M \gtrsim 6M_\odot$: directly from core contraction at end of core H burning (above S-C limit)
For $6 \gtrsim M/M\odot \gtrsim 2$: once H shell builds He core to S-C limit (non degenerate)
For $M \lesssim 2M_\odot$: once H shell builds He core to about $0.45M_\odot$ (degenerate) (end of RGB)

28.2 How to Burn Helium - the "triple alpha" process

After H burning is completed, the interior consists of ^{4}He plus the ashes of the catalytic cycle. [For stars with $M > 1M_\odot$ you have ^{14}N at an abundance equal to original CNO abundance.]

Want to put ^{4}He + ^{4}He together, looking at chart of nuclides:
http://www.nndc.bnl.gov/chart

The problem with ^{4}He burning is that ^{8}Be is unstable decays in 10^{-15} sec, goes right back to two ^{4}He.

This also makes sense in terms of binding energy. nuclear binding energy can be defined using the rest mass and mass-energy equivalence. If m is the mass of a given nucleus with N neutrons and Z protons:

$$mc^2 = Nm_nc^2 + Zm_pc^2 - BE$$

where BE is the binding energy. (sometimes the sign convention is the other way and binding energy is defined to be negative.)

In nuclear physics terms: the BE/A of ^{4}He is 7073 keV and that of ^{8}Be is 7062 keV, so that the ^{8}Be ground state is 92 keV above ^{4}He +^{4}He. i.e. ^{8}Be is less bound than 2 ^{4}He sitting next to each other, so energy must be ADDED to make the ^{8}Be. This extra energy must come from the thermal bath.

Salpeter showed that even though ^{8}Be decays, there is always a small amount of it present in the plasma at any one time. We can derive this abundance. (Then we can capture another ^{4}He onto this to make ^{12}C.) The Gamow energy for fusing the ^{4}He, is

$$E_G = (\pi\alpha 4)2m_rc^2$$

and

$$m_r = \frac{m_\alpha m_\alpha}{2m_\alpha} = \frac{1}{2}m_\alpha$$

so that $E_G = 31.4$ MeV. Use the standard form for a non-resonant reaction, then the center energy is

$$^4He +^4 He \rightarrow^8 Be: \qquad E_0 = \left(\frac{E_G(kT)^2}{4}\right)^{1/3} = 83keV(T/10^8)^{2/3}$$

So once $T > 10^8$ K or so, the COM energy becomes adequate to drive the fusion rection, since the ^{8} Be ground state is 91.8 keV above 2^4He. This is a threshold above which the reaction is fast, and the decay is fast, so there is an equilibrium. For these high T's the ^{4}He and ^{8}Be come into a nuclear equilibrium.

Since we're in equilibrium, we can just use the Saha equation, so for

$$^4He +^4 He \leftrightharpoons^8 Be$$

we have

$$\mu_4 + \mu_4 = \mu_8$$

A sideline on chemical potential μ : Just like temperature is the thermodynamic potential for entropy (e.g. heat transfer), μ is the thermodynamic potential for particles (motion, diffusion, etc). Consider the first law of thermodynamics in the form

$$dE = T\,dS - P\,dV + \mu\,dN$$

This is actually just a statement of the chain rule for an energy function $E(S, V, N)$,

$$dE = \left(\frac{\partial E}{\partial S}\right)_{V,N} dS + \left(\frac{\partial E}{\partial V}\right)_{S,N} dV + \left(\frac{\partial E}{\partial N}\right)_{S,V} dN$$

where the subscripts indicate explicitly which variables are being held fixed in the derivative. This is effectively the definition of the thermodynamic potentials T, P, and μ.

$$T = \left(\frac{\partial E}{\partial S}\right)_{V,N} \quad , \quad P = -\left(\frac{\partial E}{\partial V}\right)_{S,N} \quad , \quad \mu = \left(\frac{\partial E}{\partial N}\right)_{S,V} .$$

So just as temperature tells us how much energy wants to be moved via heat and pressure tells us how much energy is added when you squish something, chemical potential tells us the energy cost or benefit of adding / moving / removing particles. Thus we will use equal μ to impose chemical equilibrium, much like equating T gives thermal equilibrium.

28.3 How to Burn Helium - the "triple alpha" process (continued)

We need to find the equilibrium abundance of ^{8}Be,

$$\mu_4 + \mu_4 = \mu_8$$

where

$$\mu_4 = m_4 c^2 - kT \ln\left(\frac{g n_{Q,4}}{n_4}\right) \qquad \mu_8 = m_8 c^2 - kT \ln\left(\frac{g n_{Q,8}}{n_8}\right)$$

where

$$n_Q = \left(\frac{2\pi mkT}{h^2}\right)^{3/2} \sim \frac{1}{\lambda_{de}^3}$$

putting these in gives

$$\frac{n_8}{n_4^2} = \left[\frac{2h^2}{2\pi m_\alpha kT}\right]^{3/2} \exp\left[\frac{-(m_8 - 2m_4)c^2}{kT}\right]$$

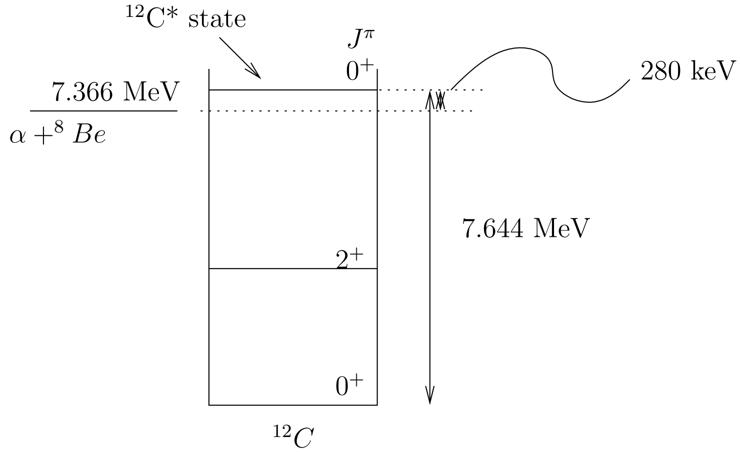
using $\rho = 10^4$ gr/cc pure helium so that $n_4 = \rho/4m_p$ then

$$\frac{n_8}{n_4} = 2.8 \times 10^{-6} T_8^{-3/2} \left(\frac{\rho}{10^4 g/cc}\right) \exp\left(\frac{-10.64}{T_8}\right)$$

where $T_8 = T/10^8$ K so at $T_8 = 2$ gives that

$$10^{-8} = \frac{n_8}{n_4}$$

now want to add another alpha



For $\alpha +^8 Be$ then $E_G = 168$ MeV. and

$$E_0 = 146 keV T_8^{2/3} = 231 keV$$

at $T = 2 \times 10^8$. The temperature in a He burning star was know but the excited state of ^{12}C was not.

Hoyle conjectured that there should be an excited state of ^{12}C that was near the Gamov window peak energy. We see from the diagram that it's there and the energy difference is 280 keV.

Note! if the resonance were at a slightly different energy, the temperature of the star would simply be different. (No anthropic monkeys here.)

The rapid reactions into ^{12}C* which usualy decays back to $\alpha +^8 Be$ allows for another chemical equilibrium. (this is a strong rection wheras the decay to the lower state of ^{12}C is electromagnetic and therefore slower.) So considering the reaction

$$^4He +^8 Be \leftrightarrow^{12} C^*$$

which might as well be

$$^4He +^4 He +^4 He \leftrightarrow^{12} C^*$$

you get

$$4\mu_4 = \mu_{12C*}$$

then saha gives

$$\frac{n_{12*}}{n_4} = 5.2 \times 10^{-10}(\rho/10^5 g/cc)^2(10^8/T)^3 \exp(-44/T_8)$$

where the -44 is given from $m_{12*}c^2 = 3m_4c^2 + 280keV$.

But the carbon can decay by emitting a photon instead of going back. The $^{12}C^* \rightarrow^{12} C$ has decay time of 1.8×10^{-13} sec. The rate of production of ^{12}C in a ground state is

$$\frac{dn_{12}}{dt} = \frac{n_{12C*}}{\tau}$$

That final step releases 7.64 MeV and that ^{12}C nucleus is now out of the chain. so the energy release is

$$erg/cc/sec = \frac{n_{12C*}}{\tau}(E)$$

putting in from the Saha equation

$$\epsilon = 5.3 \times 10^{21} \frac{ergs}{gr - sec} \left[\frac{\rho}{10^5 g/cc}\right]^2 \left(\frac{10^8}{T}\right)^3 \exp\left(\frac{-44}{T_8}\right)$$

The immediately competing rection is

$$\alpha +^{12} C \rightarrow^{16} O + \gamma$$

the fact that this occurs at the same conditions as the helium burning IS a coincidence, so that we make C+O rather than just C!

	E_G(MeV)	$E_0(T = 2 \times 10^8 K)$	$\exp[-3(E_G/4kT)^{1/3}]$
$\alpha +^{12} C$	424	315 keV	10^{-24}
$\alpha +^{16} O$	800	390 keV	10^{-37}
$\alpha +^{20} Ne$	1300	460 keV	10^{-44}

So typically a bit of the $\alpha +^{12} C \rightarrow^{16} O$ occurs and so a mix of C + O is made from He burning.

28.4 Helium Core flash ($M \leq 2M_\odot$)

For these stars, the He core increses in mass as H shell burning adds material so the star goes up the red giant branch. Refer the the bottom two figures below. The mass at which the helium core ignites. consider low mass stars first, for these He core ignites at $M = 0.45M_\odot$. When these ignite, the core is degenerate! This leads to a thermal runaway because the core doesn't change at all until you've put in enough energy to get to the Fermi energy.

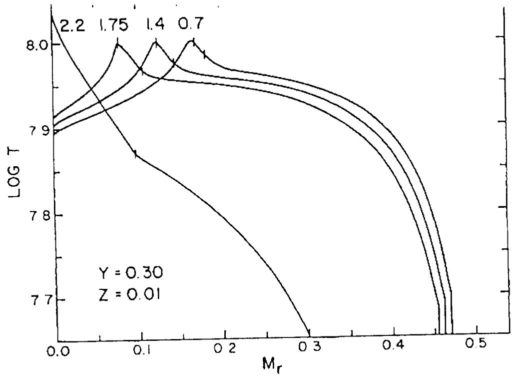


FIG. 6.—Temperature distribution within the core at the onset of the helium-core flash for red giant sequences with $(Y, Z, F_v) = (0.30, 0.01, 1.0)$. Each curve is labeled by its value of the mass M. The results shown in this figure correspond to the evolutionary phase when $L_{He} \approx 100\,L_\odot$. The tick marks along each curve denote the edges of the convective zone produced by the flash. In the 2.20 $M_\odot$ case a convective core has formed. The curves for 0.70, 1.40, and 1.75 $M_\odot$ extend out to the hydrogen-burning shell. M_r is in units of $M_\odot$.

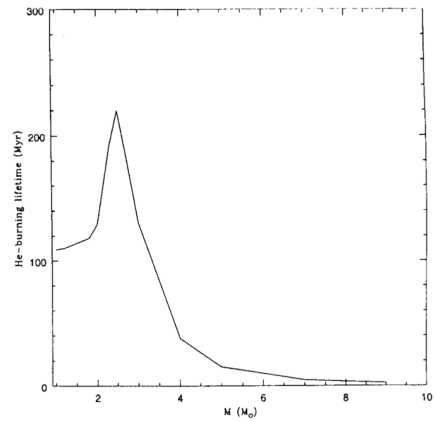
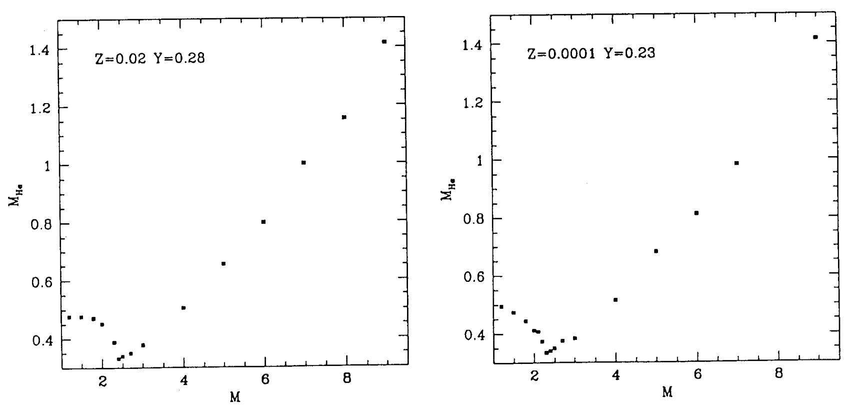


FIG. 14.—He-burning lifetime vs. the stellar mass for $Z = 0.02$ and $Y = 0.28$.



More next time...