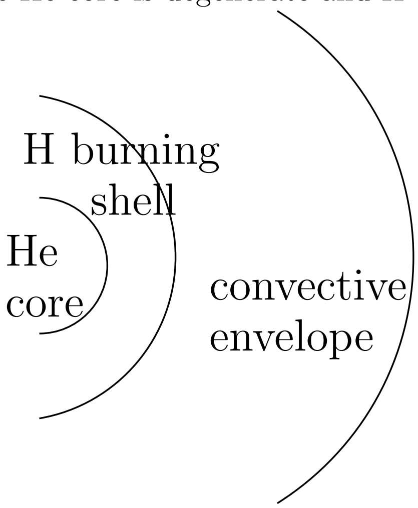
27 Stars notes 2019/11/4 - Mon - Red Giant Shell burning

27.1 Red Giant Branch Stars

The He core is degenerate and H burning is occuring in a shell above the helium core.



The radius of a degenerate object:

$$P_c \propto \frac{M^2}{R^4} \qquad P_{deg} \propto \rho^{5/3} \propto \frac{M^{5/3}}{R^5} \propto \frac{M^2}{R^4}$$

so we find

$$R \propto M^{-1/3}$$

for a non-relativistic WD as M increases R decreases. For a Relativistic object

$$P_{deg} \propto \rho^{4/3} \propto M^{4/3} R^4 \propto M^2/R^4$$

so the mass is "determined", this is related to the Chandrasekhar mass limit.

The H burning shell constantly adds He to the core and in this case sets the luminosity of the star. Very different from what we considered before. This is due to the weirdly differentiated object we're considering. The luminosity is set by the structure itself for a homogeneous star.

Roughly the condition at the bottom of the shell is that it not be geometricelly thin. The scale height of the shell is

$$H_{P,shell} = \frac{kT_s}{m_p g} = \frac{kT_s R_c^2}{m_p G M_c} \approx R_c$$

then we get

$$kT_s \simeq \frac{G M_c m_p}{R_c}$$

Need geometrically thick to have something that is thermally stable. So the shell temperature is tied directly to the size and mass of the *core*.

Now the luminosity. The convective envelope on the outside can transport anything, but the shell burning layer is radiative. Assuming the shell burning layer has thickness H_P and $H_P \approx R_c$ as above,

$$L_{rad} = 4\pi R_c^2 \left[\frac{1}{3} \frac{c}{\kappa\rho} \frac{1}{R_c} a T_s^4 \right] = \frac{4\pi}{3} \frac{R_c}{\kappa\rho_s} acT_s^4$$

Find ρ_s by demanding that

$$L_{rad} = L_{nuc}$$

where the burning is in the shell

$$L_{nuc} = \int \epsilon dM = \epsilon_{CNO}(\rho_s, T_s) 4\pi R_c^2 \times R_c \rho_s$$

note that the total mass of the star does not appear, the luminosity knows only about the core mass. Say that

$$\epsilon_{CNO}(\rho, T) = \epsilon_0 \rho T^\nu$$

then $L_{nuc} = L_{rad}$ becomes

$$\epsilon_0 \rho_s T_s^\nu 4\pi R_c^3 \rho_s = \frac{4\pi}{3} \frac{R_c}{\kappa\rho_s} acT_s^4$$

This can be solved for ρ_s

$$\rho_s = \left[\frac{1}{3} \frac{acT_s^{4-\nu}}{\epsilon_0 R_c^2 \kappa} \right]^{1/3}$$

put this back in L_{rad} to check the scaling. We get

$$L \propto R_c \left[\frac{R_c^2}{T_s^{4-\nu}} \right]^{1/3} T_s^4 \propto R_c^{5/3} T_c^{(8+\nu)/3}$$

So the luminosity depends strongly on the Helium core mass, put in $T_s \propto M_c/R_c$, then

$$L \propto \frac{1}{R_c^{1+\nu/3}} M_c^{(8+\nu)/3}$$

Since R_c just depends on the core mass this just becomes

$$L \propto M_c^{3+4\nu/9}$$

The exponent here can be 10! So it is steeply dependent on the core mass and independent of the overall mass of the star.

The fact that this doesn't depend on the mass is reflected in the HR diagrams and the T-ρ diagrams handed out. All the low mass stars shown begin to converge onto the same track at late times. "Core convergence" is seen where for all stars $< 2M_\odot$ about, the He core grows in M and

$$R_c = 2 \times 10^9 cm \left(\frac{0.1M_\odot}{M}\right)^{1/3}$$

and

$$T_s = 2 \times 10^7 \left(\frac{M}{0.1M_\odot}\right)^{4/3}$$

and

$$L = L_\odot \left(\frac{M_c}{0.1M_\odot}\right)^{11} \approx L_\odot \left(\frac{M_c}{0.16M_\odot}\right)^{7.3}$$

so the core mass grows

$$\dot{M}_c = gr/sec = \frac{L}{(E_{nuc}/m_p)} = 6 \times 10^{14} gr/sec \left(\frac{M}{0.16M_\odot}\right)^{6.3}$$

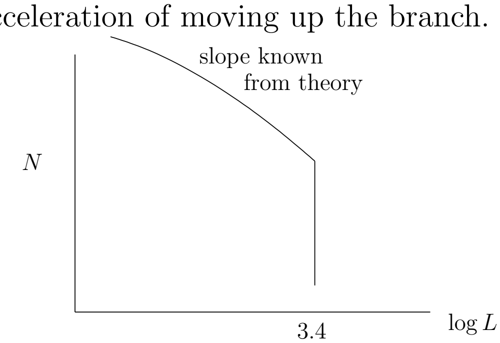
So the star spends less and less time at a given L as it ascends the red giant branch

$$t = \frac{M_c}{\dot{M}_c} = 10^{10} yrs \left(\frac{0.16M_\odot}{M_c}\right)^{6.3}$$

for $M_c = 0.25$ $t = 10^9$ yrs and $M_c = 0.4$ then $t = 53 \times 10^6$ yrs.

The helium core is getting denser $\rho \propto M^2$ and hotter because the H shell T is rising and the He core is thermally well connected. The red Giant branch evolution ends when the Helium white dwarf ignites. Which is roughly at $M_c = 0.42M_\odot$ and $L = 10^{3.2 to 3.4} L_\odot$.

Can create a histogram by counting stars at the upper end of the RGB (see color-mag diagram above) There are fewer and fewer as one moves up the giant branch. This is the acceleration of moving up the branch.



This can be used for cosmology. have to see this on top of background from massive stars. This is called the Tip of the Red Giant Branch Distance Indicator.

This is the endpoint for all stars with $M < 2M_\odot$. Can see this on the H-R diagrams above. $M > 2.2M_\odot$ don't get nearly as bright before the He core collapses when $M_c > 0.08M$.

So the compact object you make initially is a helium core. These have actually been found naked in binaries where the envelope has been transferred to the companion. Many neutron stars are seen with helium white dwarf companions.