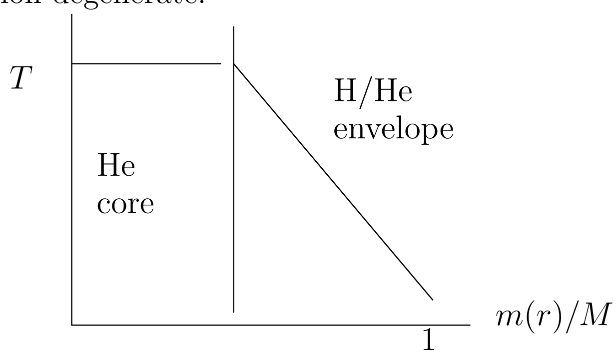
25 Stars notes 2019/10/28 - Mon - S-C limit and helium core ignition

Mention test suite and mesastar.org as sources for MESA projects. Will need to send me an email with 1-paragraph summary on due date (Nov 4).

H core burning may be followed by H shell burning to make the Helium core. But is there a limit to how big the He core can get? yes (for non-degenerate stars)

25.1 Schönberg-Chandrasekhar Limit

Consider a star after hydrogen burning in the core is done and presume that the He core is non-degenerate.



The envelope exerts pressure on the core and we want to see if there is always a hydrostatic solution. Go back to the virial theorem. Hydrostatic balance:

$$\frac{dP}{dr} = -\frac{Gm(r)\rho(r)}{r^2}$$

multiply both sides by $4\pi r^3$ and integrate (this is what we did to get the virial theorem). Let R_c be the radius of the core. integrating to here:

$$\int_0^{R_c} 4\pi r^3 \frac{dP}{dr}\, dr = 4\pi r^3 P(r)|_0^{R_c} - 12\pi \int_0^{R_c} r^2 P(r) dr$$

just integrating by parts.

$$= 4\pi R_c^3 P(R_c) - 12\pi \int_0^{R_c} r^2 P(r) dr$$

where now $P(R_c)$ is not zero, since it has the envelope above it. That's the left hand side. The right hand side is the gravitational bining energy which is roughly $-GM_c^2/R_c$. For an isothermal, ideal gas core

$$P(r) = \frac{\rho k T_c}{\mu_c m_p}$$

where μ_c is the mean molecular weight in the core (different from the envelope). putting in

$$12\pi \int r^2 \frac{\rho(r) kT_c}{\mu_c m_p} dr = 3\frac{kT_c}{\mu_c m_p} M_c$$

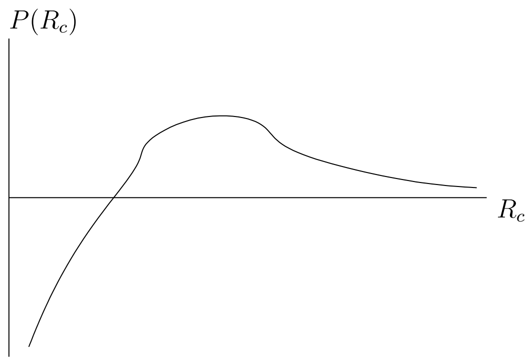
having done all the integrals, we pull it together:

$$4\pi R_c^3 P(R_c) - 3\frac{kT_c}{\mu_c m_p} M_c = -\frac{GM_c^2}{R_c}$$

Solve for $P(R_c)$,

$$P(R_c) = \frac{3}{4\pi}\frac{kT_c}{\mu_c m_p}\frac{M_c}{R_c^3} - \frac{1}{4\pi}\frac{GM_c^2}{R_c^4}$$

fixing everything and only considering dependence on radius, we see the latter term dominates at small radius and the former at large.



for fixed M_c, T_c there is a $P_{c,max}$. This gives:

$$R_{c,crit} = \frac{GM_c \mu_c m_p}{kT_c}\frac{4}{9}$$

and

$$P(R_{c,crit}) = \frac{3}{4}\frac{1}{4\pi R_c^3}\frac{M_c}{\mu_c m_p} kT_c = 0.7 \left(\frac{kT_c}{\mu_c m_p}\right)^4 \frac{1}{G^3 M_c^2}$$

(Note the temperature of the core is actually set by the Hydrogen burning shell.)

Now we need to look at the envolope. Pesume that $M = M_{env} \gg M_c$ so that

$$P_{base} = \frac{GM^2}{R^4}$$

what is R here? for the envelope: we would say that

$$kT_e = \frac{GM\mu_e m_p}{R}$$

This is that the hydrogen burning must be distributed over a scale height in order for it to be stable (for the thermostat mechanism of star to work). So then

$$P_{base} = \left(\frac{kT_e}{\mu_e m_p}\right)^4 \frac{1}{G^3 M^2}$$

To have a solution, we must have $P_{base} < P_{c,max}$. If $T_e = T_c$, then this constraint is just concerned with the relative masses and relative mean molecular weights. (don't believe any coefficients in this derivation) for the scaling we get:

$$\frac{1}{\mu_e^4 M^2} < \frac{1}{\mu_c^4 M_c^2}$$

or

$$\frac{M_c}{M} < \left(\frac{\mu_e}{\mu_c}\right)^2 (0.4)$$

for a stable solution. Here the 0.4 comes from a numerical analysis. Recall that approximately $\mu_e = 0.6$ and $\mu_c = \mu_{He} = 1.33$ and we get

$$\frac{M_c}{M} < 0.08$$

If the Helium core is non-degenerate, then $M_c < 0.08M$ for a stable hydrostatic solution.

But at lower T, core can become degenerate, which can support any pressure. So core either shrinks, raising T to start more burning, or becomes degenerate.

25.2 With Degenerate core

When the helium core becomes degenerate, then ANY M_c/M value is possible. From virial theorem:

$$4\pi R_c^3 P_c(R_c) - 12\pi \int r^2 P(r) dr = \frac{-GM_c^2}{R_c}$$

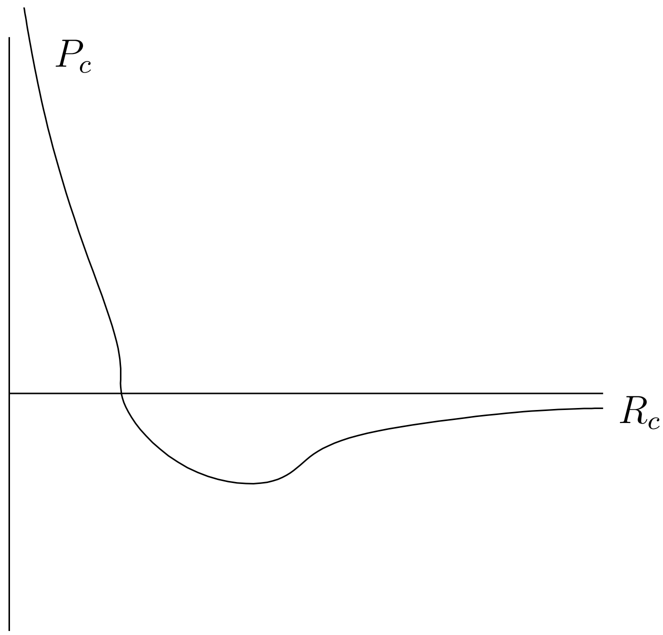
that's what we used last time. This time we'll use a degenerate equation of state. When the core is degenerate, $P(r) = K\rho^{5/3}$. Then we have

$$4\pi R_c^3 P_c = 12\pi R_c^3 K \rho_c^{5/3} - \frac{GM_c^2}{R_c}$$

but ρ goes like M/R^3, so we get

$$P_c(R_c) = (..)\frac{M_c^{5/3}}{R_c^5} - \frac{1}{4\pi}\frac{GM_c^2}{R_c^4}$$

last time the first term went like $1/R^3$ but now it goes like $1/R^5$ this time we have something that looks like



So we can accomidate any pressure, there is no maximum. So for the degenerate core, any external envelope pressure can be accomidated, and thus for a degenerate core, any value of M_c/M can exist.

25.3 Helium ignition and the core limit

Consider $M > 6M_{\odot}$. $M_c/M > 0.08$ when the H shell burning ignites. No isothermal solution is possible. (that is after burning of H runs out the star "discovers" that it can't support itself, and begins to contract) The He core contracts over a time set by the overall energy loss-rate. This is about the Kelvin-Helmholtz time,

$$t = \frac{GM^2/R}{L} \simeq 10^6 \text{ yr at } 6M_{\odot}$$

or 3×10^4 yr at $30M_{\odot}$. Refer to first figure of interior of $5M_{\odot}$ star on Kippenhanl figure and compare to previous figure. From C to D is the expansion of the giant, which happens quickly. This short-lived phase when the He core is contracting is referred to as the Hertzprung Gap. For higher mass stars core burning can ignite before reaching giant branch.

Now consider $2 < M < 6$. Then the initial value of $M_c/M < 0.08$ when H burning in core is completed. Core can be isothermal during H shell burning, and does not collapse until the H shell burning has forced $M_c > 0.08M$. This happens before the core becomes degenerate.

Finally $M < 2M_{\odot}$. Core becomes degenerate before $M_c > 0.08M$. Thus the He core can get much larger than S-C limit before the He ignites.

25.4 Evolution in density-temperature

make the plot of T_c verses ρ_c log-log. The virial theormem tells us that

$$T \propto \frac{M}{R}$$

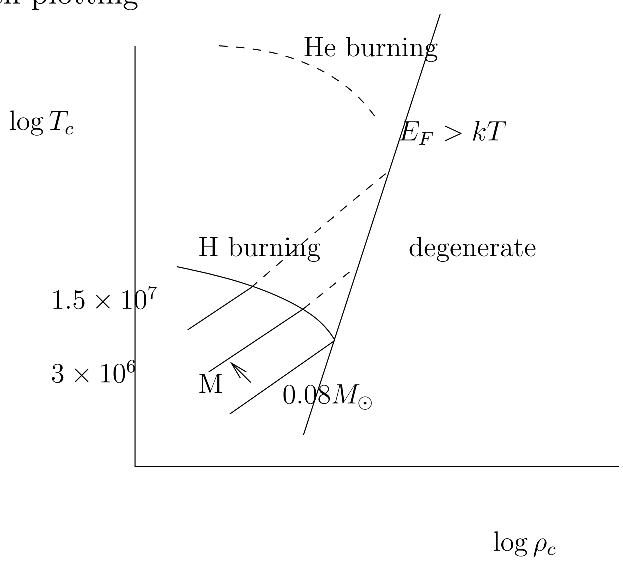
but then

$$\rho \propto \frac{M}{R^3}$$

but eliminating R gives

$$T \propto M^{2/3}\rho^{1/3}$$

then plotting



Also degeneracy is when $E_F \sim kT$, $n \propto k_F^3$ and $E_F \propto k_F^2$ this tells us that $T \propto \rho^{2/3}$ for constant degneneracy. So the final fate of the star is determined by where the curves for burning intersect the degeneracy curves. Now refer to plots that show the calculated evolution in this diagram.

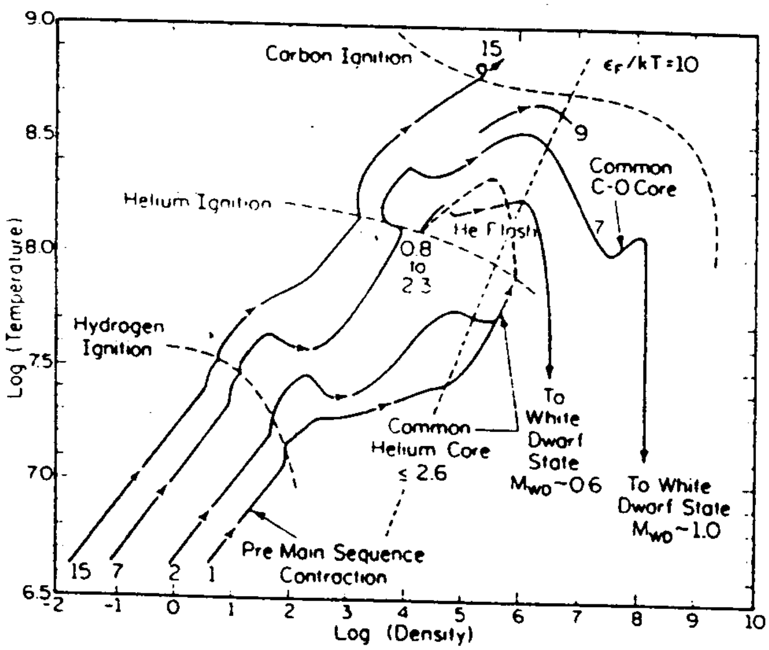


FIGURE 2.7. Central density versus central temperature for evolving stellar models. Reproduced, with permission, from I. Iben Jr. 1985, "The Life and Times of an Intermediate Mass Star," in *Quarterly Journal of the Royal Astronomical Society*, Volume 26, published by Blackwell Scientific Publications.

Also show similar figures from MESA paper (22, 29, 16, 14) and run examples of a 7 and 3 $M_\odot$ star in MESA.