21 Stars notes 2019/10/18 - Fri - CNO burning, upper MS

21.1 mesa demo continued

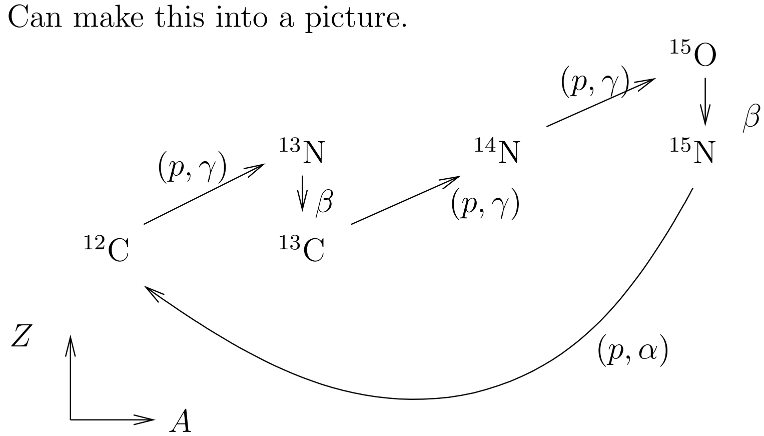
Last time showed consumption of C12 in 3 and 1 solar mass stars, and production of He3 in $1M_\odot$ star. Now show abundances in 0.4 $M_\odot$ star, which only does pp.

21.2 CN cycle

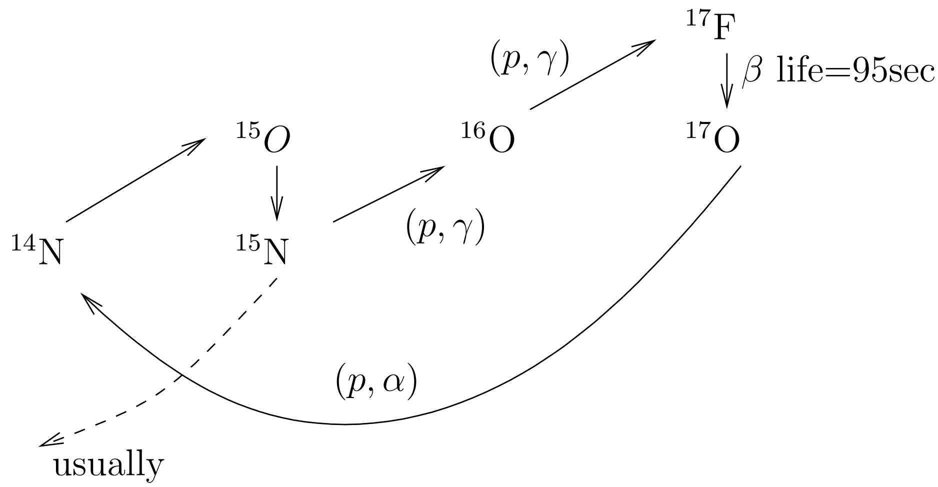
Hans Bethe and von Weizsäcer both in 1938 showed that there is a catalytic cycle involving C, N, and O. the reactions are

reaction	S (keV barn) or $t_{1/2}$ (β decay)
$p +^{12} C \rightarrow^{13} N + \gamma$	1.4
$^{13}N \rightarrow^{13} C + e^+ + \nu$	870 sec
$p +^{13} C \rightarrow^{14} N + \gamma$	5.5
$p +^{14} N \rightarrow^{15} O + \gamma$	2.8 rate limit
$^{15}O \rightarrow^{15} N + e^+ + \nu$	178 sec
$p +^{15} N \rightarrow^{12} C + \alpha$	5.3×10^4

Can make this into a picture.



Sometimes, ~ 1 part in 10^4 you get something like the following:



Because $p +^{14} N$ is the rate limiting step, in equilibrium most of the seed nuclei are in ^{14}N.

21.3 Upper main sequence and the CNO cycle

So the CN cycle "evades" the penalty of a weak interaction. By making unstable nuclei that later decay. Have to make neutrons, and this is a "better" way of doing it. The rate limiting step is just the proton capture timescale on ^{14}N. All the C N and O in the star are piled up at ^{14}N. this has $E_G = 48.1$ MeV. The rate of ^{14}N capture is then

$$n_p \langle \sigma v \rangle_{p+^{14}N \to ^{15}O+\gamma}$$

so that the energy generation

$$\epsilon = \frac{n_p n_{14} \langle \sigma v \rangle}{\rho} (28 MeV)$$

note that n_{14} is the density of "seeds" which includes all of C N and O.

For this reaction $S = 2.75$ keV barn for $p+^{14}$N and you take that $n_{seed} \simeq 10^{-3} n_p$ for solar abundance. this gives

$$\epsilon = 2.5 \times 10^{25} \frac{ergs}{gr \cdot sec} \rho \frac{1}{T_7^{2/3}} \exp \left(- \frac{72.19}{T_7^{1/3}} \right)$$

For a star on the main sequence

$$L = \int \epsilon dM_r \approx \epsilon M$$

Let's rewrite the exponential in a easier to carry form, expand about $T = 10^7$.

To expand an exponential into a power law, consider, in this case $f = e^{-72.19/T_7^{1/3}}$, which is the same as

$$\ln f = -72.19 e^{-\ln T_7^{1/3}} = -72.19 e^{-\frac{1}{3} \ln T_7}$$

or, to allow more generality,

$$\ln f = g(\ln T_7)$$

Now want to Taylor expand this around $\ln T_7 = 0$ i.e. about $T_7 = 1$.

$$\ln f \approx g(\ln T_7 = 0) + \frac{d \ln f}{d \ln T_7}(\ln T_7) = g(\ln T_7 = 0) + \nu \ln T_7$$

where we define

$$\nu = \left.\frac{d \ln f}{d \ln T}\right|_{T_7=1}$$

Re-taking the exponential of the result of Taylor expansion gives

$$f \approx \exp[g(\ln T_7 = 0) + \nu \ln T_7] = e^{g(\ln T_7=0)} \times T_7^{\nu} = f(T_7 = 1) \times T_7^{\nu}$$

In this case

$$\nu = \frac{d \ln \epsilon}{d \ln T} = \frac{1}{3}\frac{72.19}{T_7^{1/3}}$$

then approximately

$$\epsilon = 2.5 \times 10^{25} \frac{\rho}{T_7^{2/3}} \exp(-72.19) T_7^{24} \text{ erg/g-s} \approx \frac{L}{M}$$

For a given $\epsilon \approx L/M$, temperature is a very well determined quantity. Also since we'll be taking the 24th root we can just use the whole mass of the star even though the burning is very much concentrated at the center.

Using

$$L = \epsilon M = \frac{M^2}{R^3} 5 \times 10^{58} \frac{1}{T_7^{2/3}} \exp(-72.19) T_7^{24} = L_\odot M^3$$

with M and R in solar units. Then for R and M of the sun we find $T_7 = 1.85 \approx 2$. For the sun, this is not required the pp cycle can supply adequate L at lower T.

Can solve for uppor main sequence central T by putting in $R(M)$ roughly from virial argumentns

$$kT_c \simeq \frac{GMm_p\mu}{R}$$

Since we found for the sun $T_7 \approx 2$ let's re-expand at $T_7 = 2$. Then $\nu = 19$ and $f = f(T_7 = 2)(T_7/2)^{19}$

$$\frac{M^2}{R^3} 5 \times 10^{58} \frac{1}{T_7^{2/3}} (1.3 \times 10^{-25}) \left(\frac{T_7}{2}\right)^{19} = L_\odot M^3$$

Solving this equation using the above virial relation gives

$$T_7 \approx 1.83 \left(\frac{M}{M_\odot}\right)^{4/21} \approx 1.83 \left(\frac{M}{M_\odot}\right)^{1/5}$$

Thus the central temperature of a CNO burning star is nearly independent of the stellar mass.

Now to get R and $T_{\rm eff}$,

$$R \propto \frac{M}{T_c} \propto \frac{M}{M^{1/5}} \propto M^{0.8}$$

and

$$L \propto T_{eff}^4 R^2 \quad \Longrightarrow \quad T_{\rm eff}^4 \propto \frac{L}{R^2}$$

thus

$$T_{eff}^4 \propto \frac{M^3}{M^{1.6}} \propto M^{1.4}$$

so that

$$T_{eff} \propto M^{0.34}$$

so from 1-10 $M_\odot$ there is only a factor of 2 change in T_{eff} while L varies by 1000.

Refer to figures below, also MESA paper 1 fig 22 and 29. We see that this is good up to about 10 $M_\odot$. It gets modified then as you continue to go up in mass due to photons becoming important in the pressure. Note on lifetimes: The high mass stars are done before the low mass stars even start burning. very short lifetimes.

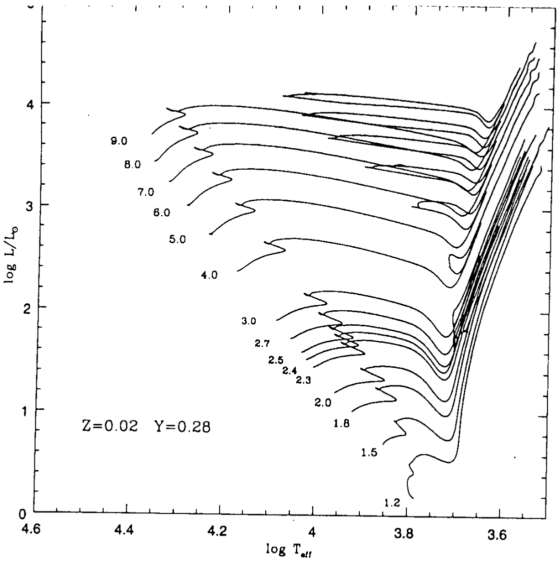


FIG. 1.—Evolutionary tracks for $Z = 0.02$, $Y = 0.28$

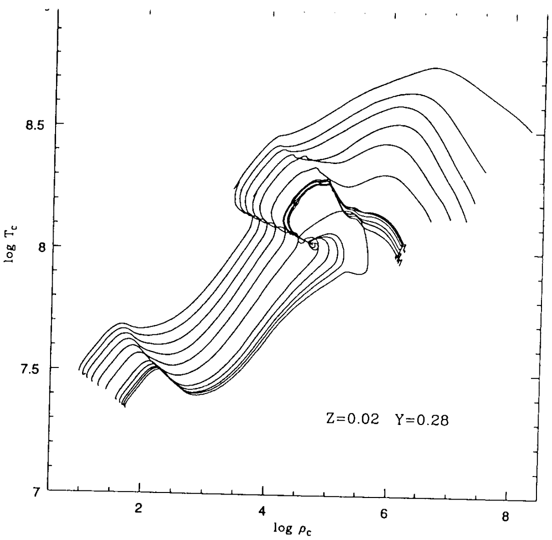


FIG. 5.—Evolution of the central temperature vs. the central density for $Z = 0.02$ and $Y = 0.28$.

PROPERTIES OF THE MODELS WITH $Z = 10^{-3}$, $Y = 0.23$

M ($M_\odot$) (1)	Δt_H (Myr) (2)	M_H^{cc} ($M_\odot$) (3)	He_{sur}^{1du} (4)	$\log L_{tip}^{RGB}$ (5)	M_{He}^1 ($M_\odot$) (6)	Δt_{He} (Myr) (7)	M_{He}^2 ($M_\odot$) (8)	He_{sur}^{2du} (9)	M_{He}^3 ($M_\odot$) (10)
1.2......	3267	0.103	0.253	3.343	0.491				
1.5......	1502	0.141	0.250	3.331	0.488	97.4	0.558	0.250	0.571
1.8......	873	0.241	0.247	3.298	0.482	95.7	0.575	0.253	0.587
2.0......	659	0.321	0.246	3.152	0.456	104	0.580	0.246	0.592
2.1......	582	0.358	0.245	2.637	0.387	144	0.564	0.245	0.579
2.2......	514	0.395	0.244	2.510	0.350	160	0.575	0.244	0.588
2.3......	462	0.433	0.244	2.402	0.339	152	0.592	0.244	0.609
2.4......	412	0.472	0.243	2.369	0.339	135	0.615	0.243	0.629
2.5......	373	0.511	0.241	2.372	0.346	118	0.630	0.241	0.644
2.7......	309	0.581	0.236	2.412	0.364	93.8	0.674	0.236	0.687
3.0......	240	0.686	0.233	2.504	0.398	68.1	0.741	0.235	0.751
4.0......	127	1.065	0.230	2.845	0.526	29.0	0.988	0.269	0.853
5.0......	78.9	1.425	0.230	3.132	0.672	16.1	1.257	0.299	0.920
6.0......	54.5	1.822	0.230	3.464	0.809	10.0	1.504	0.317	0.991
7.0......	40.6	2.202	0.230	3.597	1.013	7.2	1.776	...[a]	
9.0......	26.2	3.071	0.230	4.025	1.442	3.6	2.392	...[b]	

[a] Off-center C burning.