19 Stars notes 2019/10/14 - Mon - Pre-MS burning: Deuterium, Lithium

19.1 Exploring initial burning stages

Burning of light elements on the way to the main sequence first Deuterium, then Lithium, then reach H burning. Deuterium and Lithium are trace elements, and so cannot sustain surface luminosity for long (or at all in many cases)

Discussed figures 15, 16 from MESA paper 1 last time, now figure 17.

19.1.1 Charge hierachy

The nuclear reaction rate depends strongly on the charge of the interacting nuclei. We know that

$$\langle \sigma v \rangle = 2.6 \frac{E_G^{1/6}}{m_p^{1/2}} \frac{S(E_0)}{(kT)^{2/3}} \exp\left[-3\left(\frac{E_G}{4kT}\right)^{1/3}\right]$$

and the Gamow energy is

$$E_G = (\pi \alpha Z_1 Z_2)^2 (2 m_r c^2) = 0.988 \text{ MeV} (Z_1 Z_2)^2 \frac{m_r}{m_p}$$

as charge is increased the reaction rate goes down exponentially. For the same S (say for strong interactions). We expect purely strong reactions to preceed first amongst the lightest elements. Note that for pp the S factor is very strongly suppressed.

19.1.2 Deuterium

We want to look at how the burning proceeds, which elements are consumed first, as one moves down the Hayashi track towards the main sequence. Deuterium is the first fuel to burn.

$$p + D \rightarrow^3 He + \gamma$$

for this $S = 2.5 \times 10^{-4}$ keV barn and $m_r = \frac{1 \cdot 2}{1+2} = \frac{2}{3} m_p$ and $Z_1 = 1 = Z_2$ and then $E_G = 655$ keV and $E_{nuc} = 5.5$ MeV. The reaction rate is then:

$$\langle \sigma v \rangle = 8 \times 10^{-20} \frac{cm^3}{sec} \frac{1}{T_7^{2/3}} \exp\left(-\frac{17.24}{T_7^{1/3}}\right)$$

Let's ask when does the D burn? The lifetime of a D nucleus in the star is

$$t_D = \frac{1}{n_p \langle \sigma v \rangle} = \frac{2 \times 10^{-5} T_7^{2/3}}{\rho} \exp\left(\frac{17.24}{T_7^{1/3}}\right)$$

want to compare this to the contraction time to see when things burn

$$t_{cont} = \frac{R}{\frac{dR}{dt}} = \frac{\frac{3}{7}\frac{GM^2}{R}}{L}$$

Side note: the contraction time scale is similar to the age. This is a property of inverse power-law dependencies. i.e. it follows from

$$R(t) = R_\odot \left(\frac{\tau}{3t}\right)^{1/3}$$

Which we derived earlier, because

$$\tau_{cont} = \frac{R}{|\frac{dR}{dt}|} = \frac{(\tau/3t)^{1/3}}{|-\frac{1}{3}\tau^{1/3}/3t^{4/3}|} \sim t$$

figuring this out: on the hayashi track $T_{eff} \propto M^{7/51}L^{1/102}$ so that the effective temp is effecitively independent of L and so

$$T_{eff} \approx 4000K(M/M_\odot)^{7/51}$$

and then

$$L = 4\pi R^2 \sigma T_{eff}^4 = 9 \times 10^{32} erg/s(R/R_\odot)^2(M/M_\odot)^{28/51}$$

substitute into the contraction time above

$$t_{cont} = 1.8 \times 10^{15} s(M/M_\odot)^{3/2}(R_\odot/R)^3$$

We'd like to set $t_{cont} = t_D$. What T and ρ should we take? take the central values. The star is an $n = 3/2$ polytrope (fully convective) so:

$$\rho_c = 6\langle\rho\rangle = \frac{6M}{\frac{4}{3}\pi R^3} = 8.3\frac{g}{cm^3}(M/R^3)$$

and

$$T_c = 0.54\frac{GMm_p}{Rk_B} = 7.4 \times 10^6 K\left(\frac{M}{R}\right)$$

Where M and R in the latter expressions are in solar units. Now setting

$$t_{cont} = t_D$$

we have

$$1.8 \times 10^{15}\frac{M^{3/2}}{R^3} = \frac{2 \times 10^{-5}}{8.3M/R^3}(0.74)^{2/3}\left(\frac{M}{R}\right)^{2/3} \exp\left[\frac{17.24}{(0.74)^{1/3}}\left(\frac{R}{M}\right)^{1/3}\right]$$

or more simply

$$9.13 \times 10^{20} \frac{M^{11/6}}{R^{16/3}} = \exp\left[19.06 \left(\frac{R}{M}\right)^{1/3}\right]$$

taking logs of both sides we get

$$R = M[2.53 + 0.096 \ln M - 0.28 \ln R]^3$$

Once the star reaches this R, D will burn before futher contraction. Note that is is very nearly when $R = 20M$ in solar units, just the first term on the right, and with increasing Gamow energy (more charged nuclei) it becomes more so since the coeffficients of $\ln M$ and $\ln R$ get small. To tabulate some stuff

$M/M_\odot$	$R/R_\odot$	$T_c/10^5$ K	$0.13R/M$	t_D
0.03	0.43	5.16	1.86	7.5 Myr
0.1	1.17	6.3	1.52	1.7 Myr
0.3	2.86	7.7	1.24	0.5 Myr

19.1.3 Can D burning halt (=slow) contraction?

Need to compare the nuclear energy release to the gravitational energy. Can halt, that is significantly slow down, contraction if the nuclear energy is greater than the gravitational energy. need:

$$\frac{M}{m_p}\frac{n_D}{n_H} 5.5MeV > \frac{3}{7}\frac{GM^2}{R}$$

since $n_D/n_H \sim 2 \times 10^{-5}$. Contraction is halted if

$$M/R < 0.13$$

with M and R again in solar units. Or $0.13R/M > 1$. (see table above)

How long does this phase (Deuterium main sequence) last? The way to estimate this is to say that

$$t_{D,ms} = \frac{E_{nuc}}{L} = \frac{E_{nuc}}{E_{Grav}}\frac{E_{grav}}{L} = \left(0.13\frac{R}{M}\right) t_{contraction}$$

So then it will live for the contraction time on the deuterium burning. Note that it is a coincidence that there is actually enough energy from the deuterium to do this since it depends on the primordial abundance.

See figure. one can see the deuterium burning phase (plateu) for the 0.03 mass track. The lowest mass star for which D can burn is $\sim 0.015 M_\odot$.

Also show MESA 1 plots again figures 15-17.

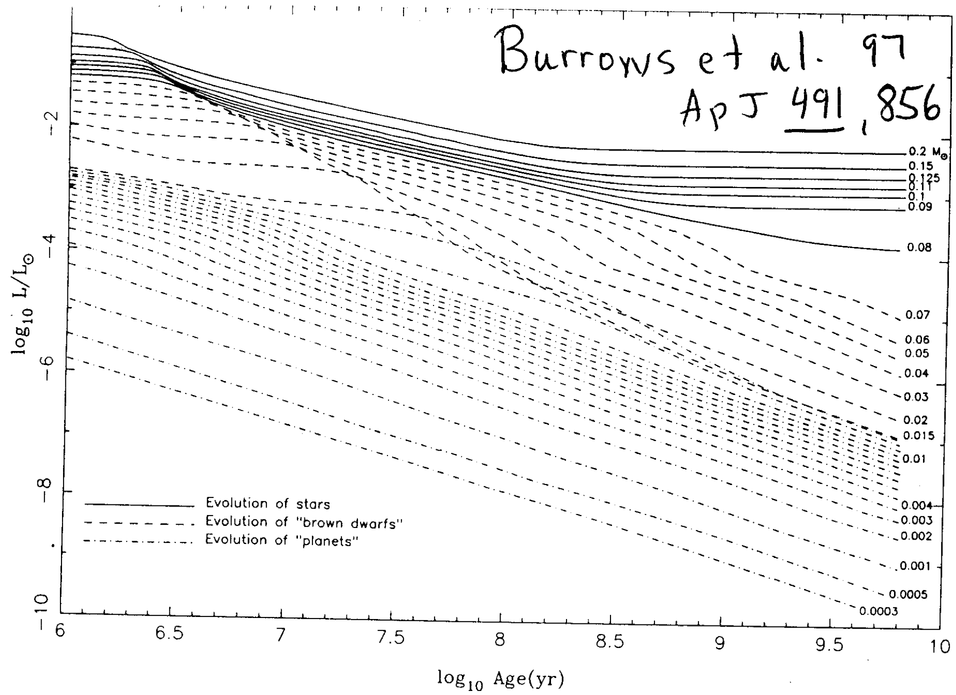

FIG. 7.—Evolution of the luminosity (in $L_\odot$) of solar-metallicity M dwarfs and substellar objects vs. time (in yr) after formation. The stars, "brown dwarfs" and "planets" are shown as solid, dashed, and dot-dashed curves, respectively. In this figure, we arbitrarily designate as "brown dwarfs" those objects that burn deuterium, while we designate those that do not as "planets." The masses (in $M_\odot$) label most of the curves, with the lowest three corresponding to the mass of Saturn, half the mass of Jupiter, and the mass of Jupiter.

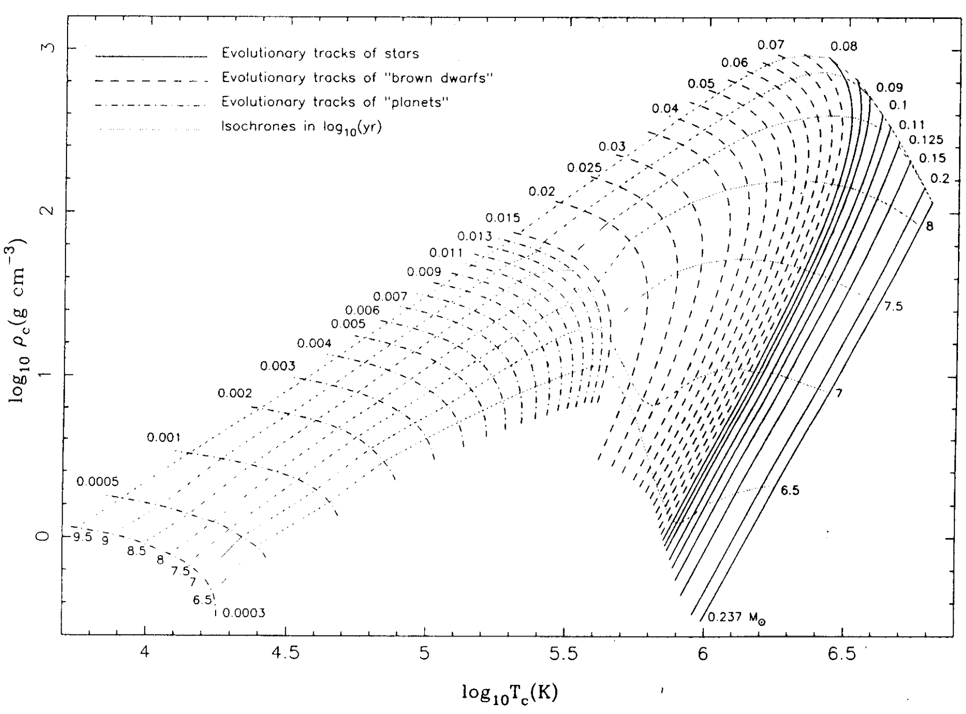

FIG. 8.—Evolutionary tracks of central density (in g cm^{-3}) vs. central temperature (in K) for stars (*solid lines*), "brown dwarfs" (*dashed lines*), and "giant planets" (*dot-dashed lines*), as in Fig. 7. The isochrones are drawn as gray curves and are labeled in $\log_{10}$ yr. The pronounced wave in the isochrones between about $\log_{10} T_c = 5.5$ and 6 is due to deuterium burning. A given mass defines a unique relationship between central temperature and density that is independent of metallicity. The only effect of the metallicity is to change the rate at which the central temperature and density evolve and the positions of the isochrones.

19.2 Lithium burning

We'll worry about

$$p +^7 Li \to 2^4 He$$

there is also

$$p +^6 Li \to^4 He +^3 He$$

but we won't consider that. For the first process $m_r = \frac{7}{8} m_p$ and $S = 120$ keV barns, and

$$E_G = (\pi \alpha Z_1 Z_2)^2 (2 m_r c^2) = 0.98 \text{ MeV} (Z_1 Z_2)^2 \frac{m_r}{m_p} = 7.73 \text{ MeV}$$

so then putting in

$$\langle \sigma v \rangle = 5.5 \times 10^{-14} \frac{cc}{s} \frac{1}{T_7^{2/3}} \exp\left(\frac{-39.27}{T_7^{1/3}} \right)$$

the higher Gamow energy means that we must go to larger temperature.

Now set $t_{Li} = t_{cont}$ then we get

$$t_{Li} = \frac{1}{n_p \langle \sigma v \rangle} = 3.3 \times 10^{-11} \frac{T_7^{3/2}}{\rho} \exp\left(\frac{39.27}{T_7^{1/3}} \right)$$

so that setting $t_{Li} = t_{cont}$ gives

$$R = M[1.419 + 4.2 \times 10^{-2} \ln M - 0.123 \ln R]^3$$

the same table as above:

$M/M_\odot$	$R/R_\odot$	$T_c/10^6$ K	t
0.08	0.26	2.3	75 Myr
0.20	0.57	2.6	27 Myr
1.0	2.29	3.2	5 Myr

This is close to the MS, but not quite. Low mass MS stars have $T_c \sim 4 \times 10^6$ K and so ther Li is depleted. Because the density of lithium is

$$\frac{n_{Li}}{n_p} \sim 10^{-9}, \qquad 10^{-4} \text{ of deuterium abundance}$$

and the energy release per reaction is similar so there is no lithium main sequence.

But Lithium provides a good clock. This is because for fully convective stars if you see lithium you can say that the core temperature is less than the values discussed above. This allows you to date the stars. See figures from Ushomirsky paper.

Ushomirsky et al. 98
ApJ 497, 253

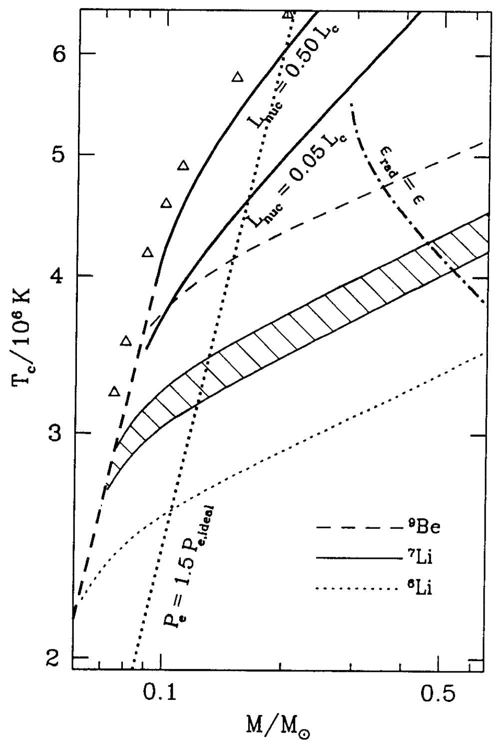

FIG. 1.—Constraints on validity of ^{7}Li, ^{6}Li, and ^{9}Be calculations. For ^{7}Li we show the temperatures at which the abundance falls from one-half to one-hundredth its initial value (*hatched region*). For ^{6}Li (*dotted line*) and ^{9}Be (*dashed line*), we plot only the curves along which one-half of the element has been depleted. We also show the constraints on applicability of our depletion calculations. Stars above the line $\epsilon_{rad} = \epsilon$ (*heavy dot-dashed line*) have radiative cores, while the curves $L_{nuc} = 0.05L_c$ and $0.50L_c$ (*heavy solid lines*) herald the arrival of the star on the main sequence. For comparison, we show central temperatures of young (1 Gyr) main-sequence stars (*triangles*) (CB97, Table 2). Coulomb corrections are unimportant *except* where the central temperature is near the maximum set by degeneracy (*heavy dashed line*), as described in § 5.1.1. The maximum temperature curve is essentially a curve of constant degeneracy, as evidenced by the comparison with the curve where the electron pressure is 1.5 times its classical value (*heavy dotted line*).

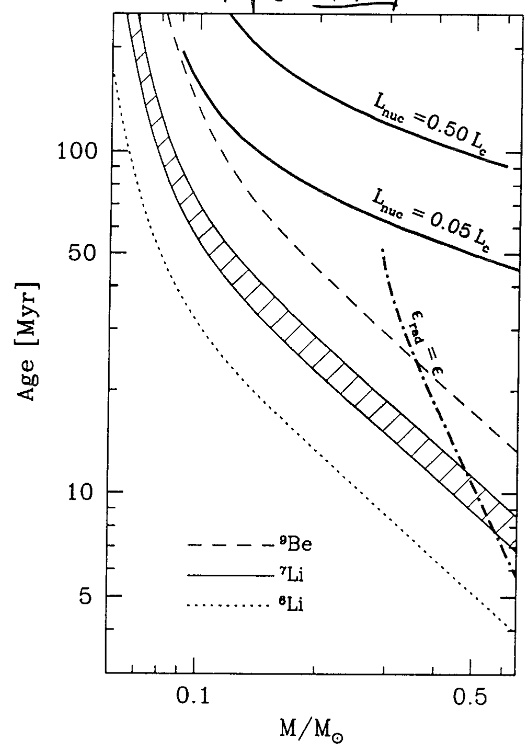

FIG. 2.—Same as Fig. 1, but for stellar age instead of central temperature. The main sequence, the maximum central temperature, and the constant degeneracy lines are not shown. The effective temperature scale $T_{eff}(M)$ is from the results of Chabrier & Baraffe (1997).

boundary curves are plotted in Figures 1 and 2 (*heavy dot-dashed lines*). The stars develop radiative cores in the regions of the plots above the $\epsilon = \epsilon_{rad}$ curves. For ^{7}Li the mass, central temperature, and density at which half-depletion is coincident with the formation of a radiative core are $M_{1/2} \approx 0.5\ M_\odot$, $T_c \approx 4.1 \times 10^6$ K, and $\rho_c \approx 4.8$ g cm^{-3}. This is in excellent agreement with CB97, who find that a radiative core forms in a 0.5 $M_\odot$ star at an age of 10^7 yr, when the lithium abundance is 0.52. The corresponding stellar parameters for which the formation of a radiative core is coincident with ^{7}Li depletion by a factor of 100 are $M_{1/100} \approx 0.44\ M_\odot$, $T_c \approx 4.3 \times 10^6$ K, and $\rho_c \approx 7.7$ g cm^{-3}. For ^{7}Li, our fully convective depletion calculations are therefore valid for $M \lesssim 0.45 - 0.5\ M_\odot$, depending on the desired depletion level.

We should point out that the above results are quite sensitive to the value of the opacity at the stellar center. If we write the opacity as $\kappa_0 \rho^{\kappa_\rho} T^{\kappa_T}$ (with ρ and T in cgs units), we find that, at the central temperature where ^{7}Li is depleted by a factor of 2, $\kappa_0 = 5.66 \times 10^{21}$ cm^2 g^{-1}, $\kappa_\rho = 0.347$, and $\kappa_T = -3.166$. The mass of the star that becomes radiative upon depleting half of its ^{7}Li content scales with κ_0 and T_{eff} as $M_{1/2} \propto \kappa_0^{0.48} T_{eff}^{1.6}$. The sensitivity of observed lithium depletions to the interior opacity has previously been considered for higher mass main-sequence stars by Swenson et al. (1990, 1994), who adjusted the interior opacities by changing the metal content. Although they were considering stars of approximately solar mass, Swenson et al. (1990) found that an increase of 37% in the interior opacity increased the mass of a Hyades star at which the lithium was half-depleted by $\sim$20%, i.e., $M_{1/2} \propto \kappa_0^{0.5}$, similar to our estimated dependence.

3. PROTON CAPTURE RATES FOR LIGHT ELEMENTS

We use thermonuclear reaction rates in the form

$$N_A \langle \sigma v \rangle = S f_{scr} T_6^{-j} \exp\left(-\frac{a}{T_6^{1/3}}\right) \text{cm}^3\ \text{s}^{-1}\ \text{g}^{-1}, \quad (15)$$

to approximate the rates of Caughlan & Fowler (1988, hereafter CF88). For nonresonant reactions, $j = \frac{2}{3}$, while for reactions affected by resonances the value of j has to be adjusted (see below). Here, S does *not* represent the astrophysical S-factor, but is rather, like a, a dimensionless