18 Stars notes 2019/10/11 - Fri - Stellar Nuclear Burning

18.1 (thermal) Nuclear reactions for star (continued)

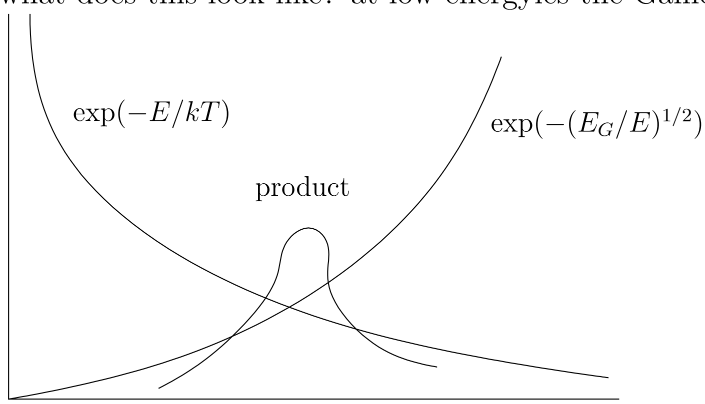
Last time we had obtained the integral:

$$\langle \sigma v \rangle = \left[\frac{m_r}{2\pi kT}\right]^{3/2} \int_0^\infty \exp\left(-\frac{m_r v_r^2}{2kT}\right) 4\pi v_r^2 dv_r \cdot v_r \sigma(v_r)$$

convert integral to E_c, center of mass energy,

$$\langle \sigma v \rangle = \frac{1}{(kT)^{3/2}} \left(\frac{8}{\pi m_r}\right)^{1/2} \int_0^\infty S(E) \exp\left(-\frac{E}{kT} - \left(\frac{E_G}{E}\right)^{1/2}\right) dE$$

so what does this look like? at low energyies the Gamov term gets you



Gamov Peak

Peaked at characteristic energy E_0.

Presume that for this integration $S(E) =$ constant, (non-resonant) the integrand is e^{-f}

$$f = \frac{E}{kT} + \left(\frac{E_G}{E}\right)^{1/2}$$

and

$$\left.\frac{\partial f}{\partial E}\right|_{E_0} = \frac{1}{kT} - \frac{1}{2}\frac{E_G^{1/2}}{E_0^{3/2}} = 0$$

so that

$$E_0^3 = \frac{1}{4} E_G (kT)^2$$

expand f around extremum

$$F(E) = f(E_0) + \frac{1}{2}(E - E_0)^2 \frac{\partial^2 f}{\partial E^2}$$

and

$$f(E_0) = 3\left(\frac{E_G}{4kT}\right)^{1/3}$$

what are some numbers:

$p+p$	$p+{}^{12}\mathrm{C}$
$E_G = 494$ keV	$= 32.6$ MeV
$E_0 = 4.5\ \mathrm{keV}(T/10^7)^{2/3}$	$= 18.2\ \mathrm{keV}(T/10^7)^{2/3}$
$\Delta/E_0 = 1T_7^{1/6}$	$= 0.5T_7^{1/6}$

Note that there can be resonaces that make the rate different in general.

Doing the integral, you get for non-resonant reactions in cgs:

$$\langle\sigma v\rangle = 2.6\frac{E_G^{1/6}}{\sqrt{m_r}}\frac{S(E_0)}{(kT)^{2/3}}\exp\left(-3\left(\frac{E_G}{4kT}\right)^{1/3}\right)$$

So we see that we are most interested in $S(E_0)$, the s-factor at this particular characteristic energy.

So if we have some reaction that releases E_{nuc} then the energy generation rate is

$$\epsilon_{\mathrm{nuc}} = \frac{n_1 n_2\langle\sigma v\rangle E_{nuc}}{\rho}$$

this has units of ergs/(gr sec) and appears in the energy equation for stellar evolution.

Note that determining $\sigma(E_0)$ experimentially is tricky, since it is at VERY low energy compared to what can be accomplished in the lab. Requires extrapolation. It is more convenient to extrapolate the S factor instead of the bare cross section. The relevant energy is shaded on the figure:

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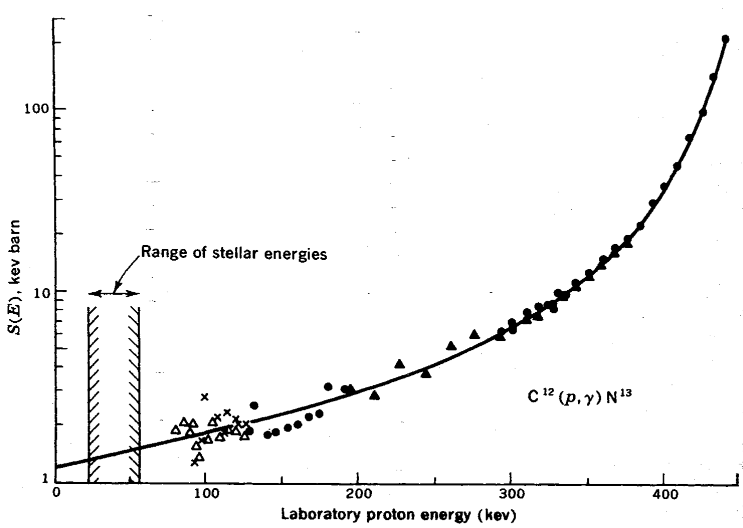

Fig. 4-5 The cross-section factor $S(E)$ for the radiative capture of protons by C^{12}. The differing types of data points represent five different experiments performed at different times and laboratories by the workers indicated. Detailed references and discussion may be found in D. F. Hebbard and J. L. Vogl, *Nucl. Phys.*, 21:652 (1960). This curve is more readily extrapolated than the one in Fig. 4-4.

18.2 Calculating for a star

Presume for *pp* cycle that the rate limiting step is $p + p \to d + e^+ + \nu_e$ now

$$\epsilon_{pp} = 2.8 \times 10^{30} \frac{S}{S_{st}} \frac{\rho}{T_7^{2/3}} \exp\left(-\frac{15.69}{T_7^{1/3}}\right) \text{ erg/g-s}$$

if we want this to halt contraction of the star then

$$L = \int \epsilon dm_r$$

Since the ϵ is so temperature sensitive, this fixes the central temperature of the star. And therefore the radius via the virial theorem.

To calculate T_c when burning matches loss. we use central temperature, so roughly

$$L = \int \epsilon dM_r = \epsilon_c M$$

For the *pp* reaction $S \sim 10^{-25} S_{strong}$. The real number is $S = 4 \times 10^{-22}$ keV-barn.

Energy balance for low mass stars Here I will do for $M \sim 0.1 M_\odot$, you should try it for the Sun using the appropriate mass-luminosity relation. For the sun you should get $T_c \sim 10^7$.

Use M-L relation for a fully convective star with free-free opacity at surface

$$L = 9 \times 10^{32} \text{ erg/s} (M/M_\odot)^{1/2} (R/R_\odot)^4$$

Also for a fully convective star, (polytrope)

$$T_c = 0.54 \frac{GM\mu m_p}{k_B R}$$

similarly we know ρ. Then you get

$$L_{nuc} = \int \epsilon dM_r = 2 \times 10^{40} \frac{m^{4/3}}{r^{7/3}} \exp\left(-\frac{1735}{m^{1/3}} r^{1/3}\right)$$

where $m = M/0.1M_\odot$ and $r = R/0.1R_\odot$ will be on the main sequence when the Ls match,

$$r = m \left[1.58 + 4.8 \times 10^{-2} \ln m - 0.36 \ln r\right]^3$$

at $0.1M_\odot \Rightarrow T_c = 3.4 \times 10^6$ K.

18.3 Exploring initial burning stages

Burning of light elements on the way to the main sequence first Deuterium, then Lithium, then reach H burning. Deuterium and Lithium are trace elements, and so cannot sustain surface luminosity for long (or at all in many cases)

Show figures 15-16 from MESA paper 1. Will discuss 17 next time.