15 Stars notes 2019/10/4 - Fri - Protostar contraction, IMF

15.0.1 pre-main-sequence contraction

Want to get an equation for the radius $R(t)$. Put $\rho \sim M/R^3$ in to get R in terms of $T ds/dt$,

$$T\frac{ds}{dt} = \epsilon - \frac{dL}{dm} \quad \text{becomes} \quad \frac{3}{2}\frac{P}{\rho}\frac{1}{R}\frac{dR}{dt} = -\frac{dL_r}{dm_r}$$

the luminosity is

$$L = -\frac{3}{2}\frac{1}{R}\frac{dR}{dt}\int \frac{P}{\rho}dm_r$$

and then

$$L = \frac{3}{2}\frac{1}{R}|\frac{dR}{dt}|\int_0^R P4\pi r^2 dr$$

Since the star is in Hydrostatic balance we can use the virial theorem to relate the integral $\int P4\pi r^2 dr$ to $(..)GM^2/R$. For a star with $P(R) \propto \rho(r)^{5/3}$ then (for this polytrope)

$$\int PdV = \frac{2}{7}\frac{GM^2}{R}$$

then the luminosity is related to the contraction rate by:

$$L = \frac{3}{7}\frac{GM^2}{R^2}\left|\frac{dR}{dt}\right|$$

Previously it was shown that

$$T_{eff} \approx 2500\left(\frac{M}{M_\odot}\right)^{1/7}\left(\frac{R}{R_\odot}\right)^{1/49}$$

this gives the luminosity is

$$L/L_\odot \approx 0.034(M/M_\odot)^{4/7}(R/R_\odot)^2$$

Combining this equation with the above we can get the time dependence. Define the variable $X = R/R_\odot$

$$\frac{dX}{dt} = -8.3\times 10^{-17}(M/M_\odot)^{4/7-14/7}X^4$$

this is

$$\frac{dX}{dt} = -\frac{X^4}{\tau}$$

where

$$\tau = 4\times 10^8 \text{ yrs}(M/M_\odot)^{10/7}$$

doing the integral

$$\int \frac{dX}{X^4} = -\int \frac{dt}{\tau} \rightarrow \frac{1}{3}\frac{1}{X_1^3} - \frac{1}{3}\frac{1}{X_2^3} = -\frac{t}{\tau}$$

the initial radius can be ignored in the general case sinciit is much larger than the final radius. i.e. a star at a time when $R_2 \ll R_1$ we get

$$R = R_\odot(\tau/3t)^{1/3}$$

this means that

$$R(t)/R_\odot = \left(\frac{1.3 \times 10^8 yr}{t}\right)^{1/3} (M/M_\odot)^{10/21}$$

this is the radius as a function of time for a fully convective contracting pre main sequence (PMS) star.

15.0.2 estimate when the star stops being fully convective

When does the fully convective solution break down. Start by looking at $M > M_\odot$. we found that

$$L = L_\odot(M/M_\odot)^3$$

where the heat transport is radiative. Radiative heat transport should start approximately when $L_{rad} > L_{Hayashi}$. When this condition is true, the surface temperature will rise above the Hayashi H^- limit. This is when

$$L_{Hayashi} = 0.034(M/M_\odot)^{4/7}(R/R_\odot)^2 < (M/M_\odot)^3$$

Put in $R(t)$ to get

$$t > 10^6 yrs \left(\frac{M}{M_\odot}\right)^{-15/7}$$

star becomes radiative after this time. Since the accretion phase is about this length, high mass stars reach the main sequence very quickly after they form.

For $M < M_\odot$

$$L_{rad} = L_\odot(M/M_\odot)^{5.5}(R_\odot/R)^{1/2}$$

so that the star becomes raditive when

$$t > 2.4 \times 10^6 yr(M/M_\odot)^{-4.4}$$

so low mass objects become radiative much later than high mass stars. This can become a hubble time for very low mass, around $0.1M_\odot$. But we haven't considered when fusion turns on yet.

For low mass stars if we say the main sequence is $R = R_\odot(M/M_\odot)$. Find that stars with $M < 0.25M_\odot$ are still fully convective while burning H on the Main sequence. Can see the signatures of the burning on the surface of such a fully convective objects.

15.1 Initial Mass Function (IMF)

This is some function you hope to get from observation $\xi(m)dm$ = number of stars born with masses between m and $m + dm$. Theorists doing fragmentation would like to predict this function. We'll talk mostly about observations. Salpeter (1955) first derived this from the observed luminosity function. He found that

$$\xi(m) \propto \frac{1}{m^{2.3}}$$

inital result $m \simeq 1M_\odot$. the total mass is then

$$M = \int \xi(m) m dm = \int \frac{dm}{m^{1.3}}$$

which diverges as you take the mass to zero. Remained in this limbo until we could measure the low mass tail.

We will use Kroupa 2001, MNRAS, 322, 231 as our reference. He plots the logarithmic distribution $\xi_L(m) = \xi(m)\, m \ln 10$, such that

$$M = \int m \xi_L(m)\, d \log_{10} m$$

This was important because people were looking for dark matter in the disk. The bottom plot demonstrates the variation in the IMF over different groups of stars.

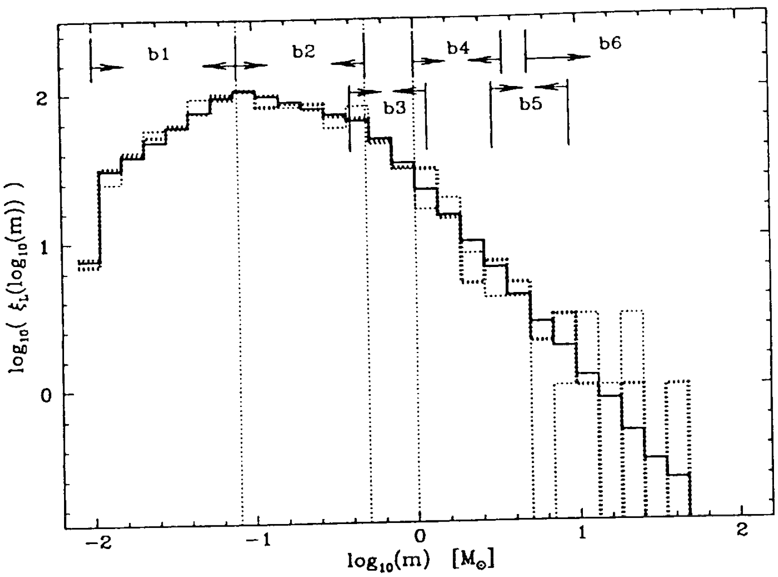

Figure 2. The adopted logarithmic IMF (equations 2 and 3), $\xi_L/10^3$, for 10^6 stars (solid histogram). Two random renditions of this IMF with 10^3 stars are shown as the heavy and thin dotted histograms. The mass ranges over which power-law functions are fitted are indicated by the arrowed six regions (equation 4), while thin vertical dotted lines indicate the masses at which α_i changes.

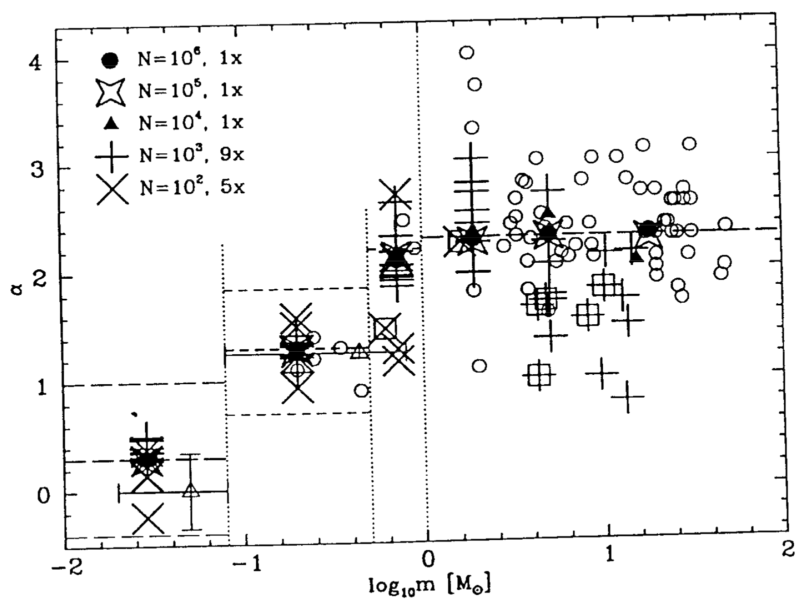

Figure 3. Purely statistical variation of α in the six mass ranges (equation 4) for different N as indicated in the key. Large outer squares indicate those α fits obtained with $nb = 2$ and 3 mass bins. The open circles, open triangles, vertical and horizontal lines are as in Fig. 1.

and parameters for these plots:

P. Kroupa, MNRAS, 322, 231 (2001)

2.2 The universal IMF

The available constraints can be conveniently summarized by the multiple-part power-law IMF (see Kroupa 2001b for details),

$$\xi(m) \propto m^{-\alpha_i} = m^{\gamma_i}, \tag{1}$$

where

$$\begin{aligned} \alpha_0 &= +0.3 \pm 0.7, & 0.01 \le m/\mathrm{M}_\odot < 0.08, \\ \alpha_1 &= +1.3 \pm 0.5, & 0.08 \le m/\mathrm{M}_\odot < 0.50, \\ \alpha_2 &= +2.3 \pm 0.3, & 0.50 \le m/\mathrm{M}_\odot < 1.00, \\ \alpha_3 &= +2.3 \pm 0.7, & 1.00 \le m/\mathrm{M}_\odot, \end{aligned} \tag{2}$$

and $\xi(m)\,\mathrm{d}m$ is the number of *single stars* in the mass interval m to $m + \mathrm{d}m$. The uncertainties correspond approximately to 99 per cent confidence intervals for $m \gtrsim 0.5\,\mathrm{M}_\odot$ (Fig. 1), and to a 95 per cent confidence interval for 0.1–0.5 $\mathrm{M}_\odot$ (KTG93). Below 0.08 $\mathrm{M}_\odot$ the confidence range is not well determined.

Note that this form differs from Scalo's (1998) recommendation, mostly because the correct structure in the luminosity function below 1 $\mathrm{M}_\odot$ is accounted for here. There is evidence for *only two changes in the power-law index*, namely near 0.5 $\mathrm{M}_\odot$ and near 0.08 $\mathrm{M}_\odot$. The frequently used Miller & Scalo (1979) IMF fails in the region 0.5–1 $\mathrm{M}_\odot$, and especially for $m \gtrsim 5\,\mathrm{M}_\odot$ (Fig. 1; see also Fig. 14 below). A useful representation of the IMF is achieved via the *logarithmic* form,

$$\xi_{\mathrm{L}}(m) = \xi(m) \ln 10 m, \tag{3}$$

where $\xi_{\mathrm{L}}\,\mathrm{d}\log_{10} m \propto m^{\Gamma_i}\,\mathrm{d}\log_{10} m = m^{-x_i}\,\mathrm{d}\log_{10} m$ is the number of stars in the logarithmic mass interval $\log_{10} m$ to $\log_{10} m + \mathrm{d}\log_{10} m$.

The adopted IMF (equation 2) has a mean stellar mass $\langle m \rangle = 0.36\,\mathrm{M}_\odot$ for stars with $0.01 \le m \le 50\,\mathrm{M}_\odot$, and leads to the following stellar population: 37 per cent BDs (0.01–0.08 $\mathrm{M}_\odot$) contributing 4.3 per cent to the stellar mass, 48 per cent M dwarfs (0.08–0.5 $\mathrm{M}_\odot$) contributing 28 per cent mass, 8.9 per cent 'K' dwarfs (0.5–1.0 $\mathrm{M}_\odot$) contributing 17 per cent mass, 5.7 per cent 'intermediate-mass (IM) stars' (1.0–8.0 $\mathrm{M}_\odot$) contributing 34 per cent mass, and 0.37 per cent 'O' stars (>8 $\mathrm{M}_\odot$) contributing 17 per cent mass.

A remarkable property of equation (2) is that 50 per cent of the mass is in stars with $0.01 \le m \le 1\,\mathrm{M}_\odot$, and 50 per cent in stars with 1–50 $\mathrm{M}_\odot$. Also, if $\alpha_4 = 0.70$ ($m > 8\,\mathrm{M}_\odot$), then 50 per cent of the mass is in stars with $8 \le m \le 50\,\mathrm{M}_\odot$, whereas $\alpha_4 = 1.4$ implies 50 per cent mass in 8–120 $\mathrm{M}_\odot$ stars. These numbers are useful for the evolution of star clusters, because supernovae (SN) lead to rapid mass-loss which can unbind a cluster if too much mass resides in the SN precursors. This is the case in clusters that have $\alpha_3 = 1.80$: stars with $8 < m \le 120\,\mathrm{M}_\odot$ contain 53 per cent of the mass in the stellar population! It is interesting that $\alpha \approx 1.8$ for $m \gtrsim 1\,\mathrm{M}_\odot$ forms the lower bound on the empirical data in Fig. 1. However, even 'normal' ($\alpha = 2.3$) star clusters suffer seriously through the evolution of their $m > 1\,\mathrm{M}_\odot$ stars (de La Fuente Marcos 1997).