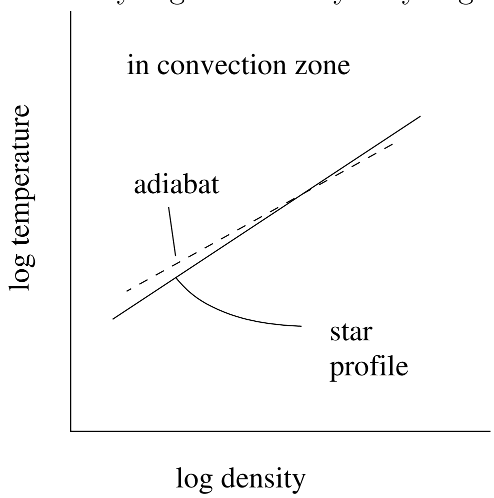
13 Stars notes 2019/09/27 - Fri - Heat transport by convection

Last time we found that velocity of convection can be quite big if $\Delta\rho/\rho \sim 1$ at a scale height. Now want to turn this around to find a realistic $\Delta\rho/\rho$.

13.1 Motion and Profile in Unstable Case (continued)

What is gradient required to carry flux? Our aim is to figure out how big $\Delta\rho/\rho$ needs to be to carry a useful flux. This sets the temperature profile.

Generally a gradient only very slightly more than the adiabatic one is required.



What flux will convection carry when $\Delta\rho/\rho|_{\ell=H} \ll 1$. for transport

$$Flux = v(\text{energy density})$$

the energy density is the kinetic energy so then when efficient,

$$F_{conv} \simeq \rho v_c^3$$

so then with $v = \sqrt{g\ell\Delta\rho/\rho}$

$$F = \rho(g\ell)^{3/2}\left(\frac{\Delta\rho}{\rho}|_{\ell=H}\right)^{3/2}$$

then with $\ell = H = kT/m_p g$ gives

$$F_{conv} = \rho\frac{kT}{m_p}\left(\frac{kT}{m_p}\right)^{1/2}\left(\frac{\Delta\rho}{\rho}\right)^{3/2} = Pc_s\left(\frac{\Delta\rho}{\rho}|_{\ell=H}\right)^{3/2}$$

note we still havent evaluated the density contrast.

We'd like to compare this to something, to set flux scale,

$$F_{rad} \simeq \frac{1}{3}\frac{c}{\kappa\rho}\frac{d}{dr}aT^4 = \frac{P_{rad}c}{\tau}$$

where $\tau = \kappa\rho R$. so then taking the ratio

$$\frac{F_{conv}}{F_{rad}} = \frac{c_s}{c}\frac{P}{P_{rad}}\tau\left(\frac{\Delta\rho}{\rho}|_{\ell=H}\right)^{3/2}$$

Outer Part (surface of the sun) putting in the values $P/P_r \approx 10^4$ and $T = 10^4$ K, which gives $c_s \approx 10^6$ cm/s ($c = 3 \times 10^{10}$cm/s). then

$$\frac{F_{conv}}{F_{rad}} \approx 1 \quad \Rightarrow \quad \frac{\Delta\rho}{\rho} \approx \left[\frac{c}{c_s}\frac{P_{rad}}{P}\frac{1}{\tau}\right]^{2/3} \approx \frac{1}{\tau^{2/3}}$$

In the outer parts of stars, where $\tau < 100$ or so, if convection is occurring it is vigorous implies that $\Delta\rho/\rho \sim 1$ and $v_c \sim c_s$. One has to do a case like this numerically really because the approximations we are talking about here fall apart.

Now in the interior $\tau \ggg 1$ then

$$\frac{F_{conv}}{F_{rad}} \approx \frac{c_s}{c}\frac{P}{P_{\rm rad}}\tau\left(\frac{\Delta\rho}{\rho}\right)^{3/2} \simeq \frac{c_s}{R}\left[\frac{P}{P_{rad}}\tau\frac{R}{c}\right]\left(\frac{\Delta\rho}{\rho}\right)^{3/2}$$

note that $R/c_s = t_{dyn}$, the thing in [] is actually the Kelvin-Helmholtz (thermal) time. Note that the pressure factor is there in front of the random walk time because most of the energy is not in the photons. so we get

$$\frac{F_{conv}}{F_{rad}} \simeq \frac{t_{KH}}{t_{dyn}}\left(\frac{\Delta\rho}{\rho}\right)^{3/2} \quad \text{for sun} \quad \approx \frac{10^7 \text{ years}}{1 \text{ hour}}\left(\frac{\Delta\rho}{\rho}\right)^{3/2} \approx 10^{11}\left(\frac{\Delta\rho}{\rho}\right)^{3/2}$$

really big (times the density contrast). This is implies that for convection to be efficient in a stellar interior it only needs

$$\frac{\Delta\rho}{\rho} \sim \left(\frac{t_{dyn}}{t_{KH}}\right)^{2/3} \sim 10^{-7} \ll 1$$

this implies that the profile within the star is very nearly the adiabatic relation. So for a fully convective star

$$\frac{d\ln T}{d\ln P}|_* = \frac{2}{5}$$

and then

$$T(r) \propto P(r)^{2/5} \propto \rho^{2/5}T^{2/5}$$

so that

$$P(r) \propto \rho(r)^{5/3}$$

and

$$T(r) \propto \rho(r)^{2/3}$$

Solar dels again:

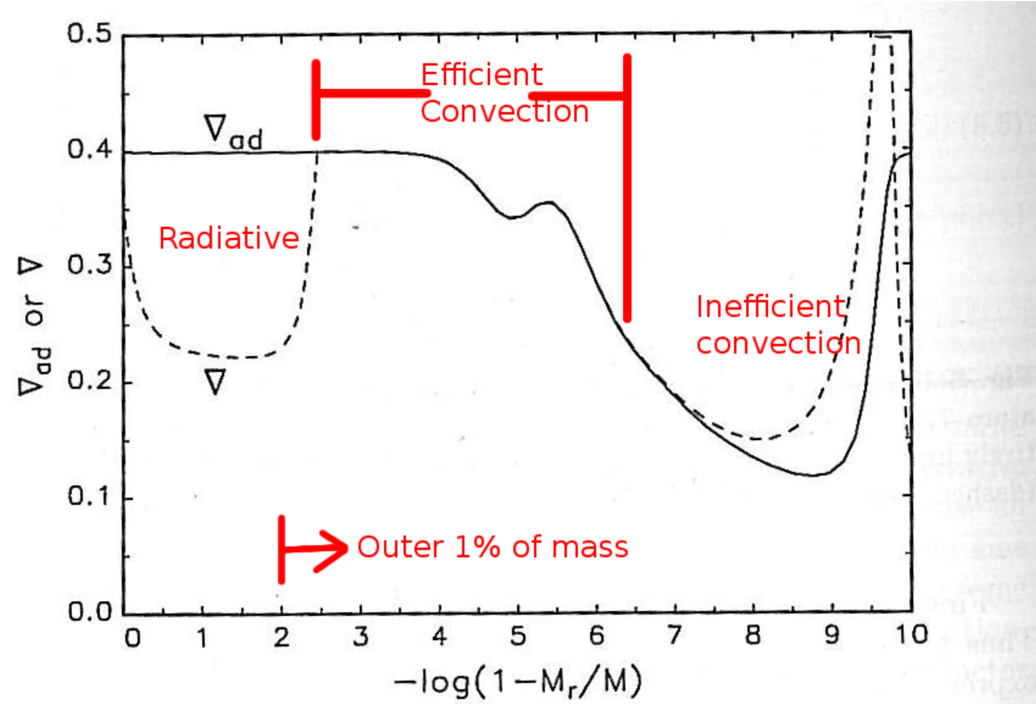


Fig. 5.2. Shown are ∇_{ad} (solid line) and ∇ (dashed line) versus $-\log(1-\mathcal{M}_r/\mathcal{M})$ for a model ZAMS sun. See also Fig. 3.11.

Show MESA models again 3 and then 1 Msun if time.