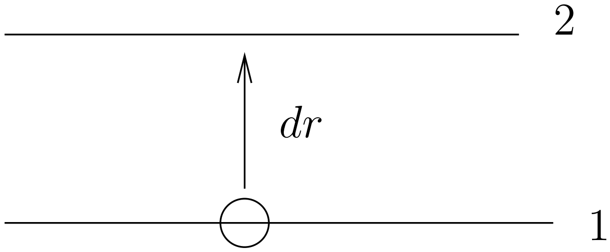
11 Stars notes 2019/09/18 - Wed - Convection

11.1 Starting Convection

So far we have only discussed electron conduction or radiative transport as the mode of transport for heat within the star. Sometimes the required temperature gradient ∇T you need is so steep that convection occurs. We want to derive the condition for the onset of convection and how efficient this form of transport can be.

Show some convection zones in stars with MESA. $3M_\odot$ and $1M_\odot$ from to central H fraction of 0.2. Also change stopping condition to central helium and the $1M_\odot$ case will go on to the first giant phase, which has a large outer convection zone.

Consider a fluid element at position 1. Take the element to be well insulated, so that it is completely adiabatic. Now push this up a distance dr to position 2.



now

$$TdS = dE + PdV = 0$$

for the perturbed element. First we will say that we allow the fluid element to come into pressure equlibrium with its surroundings at position 2. When you push it up the density will decrease as the pressure falls. If the resulting density is less than the surrounding, the element continues to rise, i.e. instability.

Presume adabatic so that $PV^\gamma = const$ or $P/\rho^\gamma = const$ so that for the bubble

$$\rho_2 = \rho_1 \left(\frac{P_2}{P_1}\right)^{1/\gamma}$$

where γ is the adiabatic index for the fluid. $\gamma = 5/3$ for monatomic ideal gas. We have a stable situation when $\underline{\rho_{2,bub} > \rho_{2,*}}$ so for stability:

$$\left(\frac{P_2}{P_1}\right)^{1/\gamma} \rho_1 > \rho_2$$

where now all the indices are for the star. We can make this into a function, where the local pressure determines the density of the adiabatically moving "bubble" as it moves:

$$\rho_{\rm bub}(r) = \rho_1 \left(\frac{P(r)}{P_1}\right)^{1/\gamma}$$

so that

$$\frac{d\rho_{\rm bub}}{dr} = \frac{\rho_1}{P_1^{1/\gamma}} \frac{1}{\gamma} P^{1/\gamma - 1} \frac{dP}{dr} = \rho_1 \frac{P^{1/\gamma}}{P_1^{1/\gamma}} \frac{1}{\gamma} \frac{1}{P} \frac{dP}{dr} = \frac{1}{\gamma} \frac{\rho}{P} \frac{dP}{dr} \quad .$$

or

$$\frac{d\ln\rho_{bub}}{dr} = \frac{1}{\gamma}\frac{d\ln P}{dr}$$

Note $d\rho/dr$ is negative, so we need to careful with the inequality. In order to be stable, the density of the moving bubble should not go down as fast as the background density goes down with increasing radius. Thus the derivative for the bubble should not be as negative, or

$$\frac{d\ln\rho_{\rm bub}}{dr} > \frac{d\ln\rho}{dr}$$

implies stability.