10 Stars notes 2019/09/13 - Fri - Photospheres

10.1 Photospheres

and working with a Plane-Parallel atmosphere
Want to derive the "anisotropy" in the radiaton field. The flux is F in ergs/cm^2 sec. Flux in radiation field is something like $Ec = aT^4c$. We want to compare these so we want

$$\text{anisotropy} \sim \frac{F}{acT^4}$$

can measure the flux with temperature: $F = \sigma_{SB}T_{eff}^4$. Then modulo constants,

$$\frac{F}{acT^4} \approx \frac{T_{eff}^4}{T^4}$$

for the sun with $T_{eff} \simeq 6000$ K and $T \sim 10^7$ this is a small number.

But let's do it

$$F = -\frac{1}{3}c\frac{1}{\kappa\rho}\frac{d}{dz}aT^4 = \frac{ac}{3\kappa}\frac{d}{dy}T^4$$

F =constant near the surface, since energy is not released there, just flowing through, also κ =constant so define the optical depth via $d\tau = \kappa dy$. and so

$$\tau = \text{optical depth} = \int_{outside}^{in} \kappa dy$$

photons can penetrate to $\tau \sim 1$. now

$$F = \frac{ac}{3}\frac{d}{d\tau}T^4$$

and integrating

$$\int_0^\tau F d\tau = \int_{surface}^T \frac{ac}{3} dT^4 \quad \Longrightarrow \quad F\tau \approx \frac{ac}{3}T^4$$

so for high optical depth $\tau \gg 1$

$$F = \frac{1}{\tau}\frac{ac}{3}T^4(r)$$

This is called the Radiative "zero" solution, you neglect the surface condition, is good when $T(r) \gg T_{surf}$.

Then with

$$F = \sigma_{SB}T_{eff}^4 = \frac{1}{3}\frac{ac}{\tau}T^4$$

so the anisotropy

$$\text{anisotropy} \sim \frac{T_{eff}^4}{T^4} \sim \frac{1}{\tau}$$

for $\tau \gg 1$ the radiation field in the star is nearly isotropic. Define the "photosphere" as the place where departing photons last scattered. This is the surface for our purposes (what you see). $\tau \approx 1$. roughly, $T_{eff} = T$.

How do we estimate P at the photosphere?

We know $T = T_{eff}$. Want to get the density from τ. First a qualitative estimate:

$$\tau = \int \kappa dy = \int \kappa \rho dz$$

or

$$= \int \frac{\sigma}{m_p} \rho dz = \int \sigma n_e dz = \boxed{\tau = \int \frac{dz}{\ell_\gamma}}$$

so at $\tau = 1$ is where the photon mean free path is comparable to the scale height, $\ell \simeq dz \simeq H$.

Now a calculation

$$\tau = 1 = \kappa \int \rho dz = \kappa y$$

from hydrostatic balance

$$\frac{dP}{dz} = -\rho g$$

but at the surface $g = GM/R^2$ is constant and

$$\int dP = -g \int \rho dz = gy = P$$

so then $\tau = 1$ implies that

$$\int \rho dz = y = \frac{1}{\kappa}$$

so then

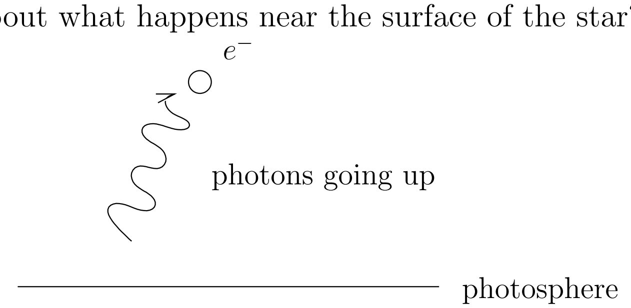
$$\boxed{P_{ph} = g\frac{1}{\kappa}}$$

Similar for non-constant opacity, just harder integrals.

10.2 The Eddington luminosity

A physical picture

So far all we've shown is that when $L \simeq L_{edd}$ radiation pressure becomes dominant. What about what happens near the surface of the star?



The photons will push the electron up. The rate of photons hitting the electron comes from

$$r_{coll} = (n_\gamma c)(\sigma)$$

but the electrons gets $\Delta p \simeq E_\gamma / c$. So that the force on the electron

$$F = \Delta p r_{coll} = \sigma n_\gamma c \frac{E_\gamma}{c} = \frac{\sigma}{c}(n_\gamma c E_\gamma) = \frac{\sigma}{c}(\text{Flux})$$

since the flux near the photosphere (free streaming photons) is $n_\gamma E_\gamma c$. We want to compare this to the force on a proton $F_{proton} = m_p g$. Trouble when

$$\frac{\sigma}{c} \frac{L}{4\pi r^2} > m_p \frac{GM}{r^2}$$

or when

$$L > \frac{4\pi G M m_p c}{\sigma}$$

which is again the Eddington luminosity if $\sigma = \sigma_{Th}$. Massive stars can really shed a lot of matter in this way.