7 Stars notes 2019/09/06 - Fri - Energy budget, heat transport

7.1 Simplest gravithermal balance

Using the virial relation we can express the basic thermal and physical structure of the star in one line. Start from

$$E_{grav} = -3V\langle P\rangle \sim -\langle P\rangle V = -\frac{kT}{m_p}\rho V = -kT\frac{M}{m_p}$$

also, we defined

$$E_{Gr} = -\int \frac{Gm(r)dm}{r} \simeq -\frac{GM^2}{R}(..)$$

so if a star is losing energy then the radius must be decreasing. Putting these together gives a relation that expresses the star's basic structural properties,

$$\boxed{kT \sim \frac{GMm_p}{R}}$$

This is roughly the temperature at the half mass point. for the sun this is like 10^7 K. Note again that as the star contracts, T goes up.

7.2 The classic stellar energy budget paradox

The problem: Today the luminosity of the sun is $L = L_\odot = 4\times10^{33}$erg/sec. $E_{Gr} = 4\times10^{48}$ ergs. So if there were no internal heat source, how long would it take for E_{Gr} to change by a factor of 2? this is the **Thermal time** also known as the **Kelvin-Helmholtz time**

$$t_{K-H} = \frac{E_{KE}}{L} \approx \frac{-E_{GR}}{L} \approx \frac{GM_\odot^2}{R_\odot^2 L_\odot} = 10^{15}\text{ sec} = \text{few}\times10^7\text{ years} \ .$$

But the Earth is much older than this. So the sun can't be living in it's current state on gravitational energy. This was a big scientific driver of the discovery of nuclear energy.

What about chemical energy? What is the gravitational energy per mass

$$\frac{E_{grav}}{M} = \frac{GM}{R} = 10^{15} ergs/g$$

chemistry

$$\frac{5eV}{m_p} = 5\times10^{12} ergs/gr$$

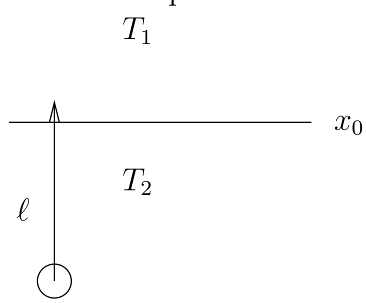
nuclear

$$\frac{7MeV}{m_p} = 7\times10^{18} ergs/gr$$

which allows the sun to live a factor of 1000 longer which is about a 3 billion years.

7.3 Heat transport - thermal structure of star

Stars are moving heat out to their environment, we'd like to describe how this functions. We are used to conduction by electrons (like a metal) but in a star the dominant effect is actually radiative transport via photons, but not the free streaming type you may be used to, diffusion of photons.



We'll start from thinking about conduction. Think of an interface with T_2 below and $T_1 < T_2$ above. Now consider E as the internal energy per unit volume of the particles which are transporting the energy. ℓ is some mean free path to a collision. for a simple cross section $\ell = 1/\sigma n$ where σ is the cross section, n is the number density of scatterers and v is the velocity of the transporting particles. Can think of electrons in a metal (Drude model) or photons or electrons in a gas.

Want to find the flux of energy crossing the interface, F. will have units of ergs/(cm^2 sec). Particles moving up across the boundary carry excess energy upward, wheras those moving down carry a lack of energy down. Particles crossing the boundary were generally scattered a distance ℓ away from the boundary. So the flux down is (with interface at x_0)

$$F_{down} = \frac{1}{6} v E(x_0 + \ell)$$

and

$$F_{up} = \frac{1}{6} v E(x_0 - \ell)$$

then with $E(x_0 + \ell) = E(x_0) + \frac{\partial E}{\partial x}\,\ell$ say that

$$F_{up} - F_{down} = F_x = -\frac{1}{3} v\ell \,\frac{dE}{dx}$$

So this is a general form, now we go on to some particular cases: using

$$\frac{dE}{dx} = \frac{dE}{dT}\,\frac{dT}{dx}$$

Ideal gas:

$$\frac{dE}{dT} = \frac{3}{2} n k_B$$

so that

$$F_x = -\frac{1}{3} v\ell \frac{3}{2} n k_B \,\frac{dT}{dx}$$

the coefficient here is the thermal conductivity. The mean free path is

$$\ell = \frac{1}{n_S \sigma_S}$$

assuming that the number of scatterers is the same as the number of carriers, in plasma with $n_e = n_p$ then you get

$$F = -\frac{1}{2} v \frac{k_B}{\sigma_S} \frac{dT}{dx}$$

one still has to do integrals to get the v right.

7.3.1 electron conduction

(will get to the right answer, photon diffusion, in a moment)

Now let's see if this can explain the solar luminosity.
i.e. can electron conduction transport heat at $L = L_\odot$? first need to estimate the cross section for electron proton scattering. A good estimate is given by the turning point for thermal electrons in the coulomb potential, then

$$\frac{e^2}{r_{90^\circ}} \simeq kT$$

and this gives

$$\sigma_{e-p} \simeq \frac{e^4}{(kT)^2}$$

Then for getting a rough estimate, dropping factors of 2 and 3

$$F = v \frac{1}{\sigma} k \frac{dT}{dx} \approx \left(\frac{kT}{m_e}\right)^{1/2} \frac{(kT)^2}{e^4} \frac{kT}{R}$$

so we're drawing a circle halfway through the star and evaluating for that. This gives

$$L = 4\pi R^2 F = 4\pi R \frac{(k_B T)^{7/2}}{m_e^{1/2} e^4} = 5 \times 10^{31} \text{ erg/s} \left(\frac{T}{10^7 \text{ K}}\right)^{7/2} .$$

This is about 2 orders of magnitude too low for the sun ($L_\odot = 4 \times 10^{33}$ erg/s). So this conduction can't do it.

Protons are even worse.