Theoretical Astrophysics Homework: 2.b Light and Matter

Due: September 26

Problem 1:
Carroll & Ostlie Problem 8.4

Problem 2:
What fraction of neutral hydrogen atoms are in the $n = 2$ level at a temperature of 9,000 K? (assume $n \approx n_1$. That is, the fraction in the $n = 2$ state is small.)

Problem 3:
To understand the relative importance of the different parameters in the Saha equation, perform the following exercises. First assume that $T = 6000K$, $n_e = 1015$ cm^{-3}, and the ionization potential = 12 eV. By what factor does the ionization ratio (N_+/N_0) change when we separately: (a) double the temperature? (note you can do $[(N_+/N_0)_{T_2}/(N_+/N_0)_{T_1}]$) (b) double the electron density? (c) double the ionization potential? Which was more important during the temperature change, the exponential term or the $T^{3/2}$ term?

Problem 4:
Find the number density of neutral hydrogen atoms in a pure hydrogen gas at 8,000 K if the number of electrons is 0.3 cm^{-3} (interstellar intercloud medium like conditions). Next estimate the number density of neutral hydrogen atoms in a cold 100 K hydrogen cloud in pressure balance with the intercloud medium.

Additional problems for AY 521

Problem 5:
Absorption lines are observed at 614.2, 580.4, 561.7, and 550.3 nm. It is believed these are from hydrogen. Which series are they from, what level transitions are they, and what is the radial velocity implied by the doppler shift?

Problem 6:
Suppose absorption lines from Ca XV (14 times ionized) are twice as strong as absorption lines from Ca XIII (12 times ionized) and the line strengths are directly proportional to the ratio of the number densities $n_{\rm XV}/n_{\rm XIII} = n_{14}/n_{12}$.

(a) Use the Saha equation to show that

$$\frac{n_{14}}{n_{12}} n_e^2 = 4 \frac{g_{14}}{g_{12}} \frac{(2\pi m_e kT)^3}{h^6} e^{-(\chi_{13}+\chi_{12})/kT}$$

where, for example, χ_{12} is the ionization energy from the 12 times ionized state to the 13 times ionized state.

(b) Find the electron density and electron pressure if $T = 5 \times 10^5$ K. Assume $g_{14}/g_{12} = 1$, $\chi_{13} = 727$ eV, and $\chi_{12} = 657$ eV