38 Astro notes 2018/11/30 - Fri - Cosmology - dynamics, fate

38.1 Dynamics of the expansion factor (continued)

Last time I stated the differential equation for the expansion history $R(t)$ is the Friedmann equation

$$\left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho - \frac{1}{3}\Lambda c^2\right] R^2 = -kc^2$$

Where the mass-energy has several components (note that Λ can be considered either a density component or a separate constant.)

$$\rho_m \propto R^{-3}$$

$$\rho_{rel} \propto R^{-4}$$

$$\rho_\Lambda = constant$$

Since different components dominated the dynamics at different times, there are three "epochs" of expansion: an early radiation-dominated-era, a matter dominated era, and we are now moving into an era dominated by what appears to be vacuum energy or cosmological constant.

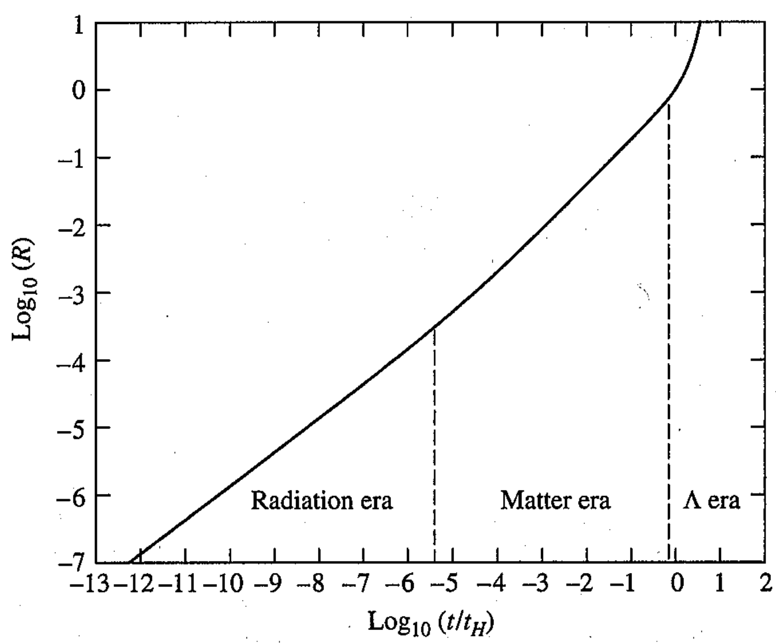

FIGURE 29.19 A logarithmic graph of the scale factor R as a function of time. During the radiation era, $R \propto t^{1/2}$; during the matter era, $R \propto t^{2/3}$; and during the Λ era, R grows exponentially.

or on a linear scale

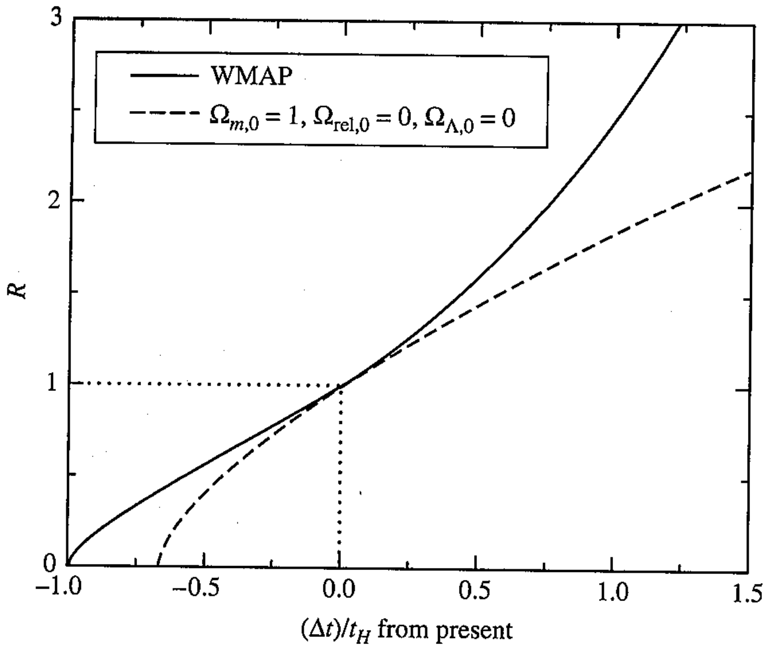

FIGURE 29.20 The scale factor R as a function of time, measured from the present, for a WMAP universe with $t_0 \simeq t_H$, and a flat, one-component universe of pressureless dust with $t_0 = 2t_H/3$ (Eq. 29.44). The dotted lines locate the position of today's universe on the two curves.

38.2 Cosmological Parameters

With the structure of the homogeneus, isotropic universe specified by the Robertson-Walker metric, we are now discussing how the scale factor $R(t)$ varies in time and how it is related to the curvature k. We found the evolution of $R = 1/(1+z)$ is given by:

$$\left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G(\rho_m + \rho_{rel}) - \frac{1}{3}\Lambda c^2\right] R^2 = -kc^2$$

Recall that the Hubble parameter is defined such that $H = \dot{R}/R$.

We can divide all mass-energy density components by

$$\rho_c = \frac{3H^2}{8\pi G}$$

which is the so-called "critical" density. One can see that if $\Lambda = 0$ and $\rho = \rho_c$, from the definition of H, $k = 0$. This was called "critically expanding" universe before the cosmological constant was discovered, since it was the border between matter-dominated solutions that recollapse and those that expand forever. With these parameters it is possible to simplify the Friedmann equation to

$$H^2[1 - (\Omega_m + \Omega_{rel} + \Omega_\Lambda)]R^2 = -kc^2$$

Where

$$\Omega_i = \frac{\rho_i}{\rho_c}$$

and for dark energy

$$\Omega_\Lambda = \frac{\Lambda c^2}{3H^2}$$

Then the densities can be written like so:

$$\rho_m = \Omega_m \rho_c$$

so that, for example,
(Student: write $\rho_m(R)$)

$$\rho_{m,0} = \Omega_{m,0}\rho_{c,0} \quad \text{and} \quad \rho_m = \rho_{m,0}R^{-3} = \Omega_{m,0}\rho_{c,0}R^{-3} = \Omega_{m,0}\frac{3H_0^2}{8\pi G}R^{-3}$$

and similarly for ρ_{rel} with R^{-4}.

The above dynamics can be solved for $H = \dot{R}/R$ in terms of the Ω's where $\Omega = \Omega_m + \Omega_{rel} + \Omega_\Lambda$, to give an expression for the expansion rate history of the universe in terms of the current values of the component fractions:

$$H = H_0(1+z)\left[\Omega_{m,0}(1+z) + \Omega_{rel,0}(1+z)^2 + \frac{\Omega_{\Lambda,0}}{(1+z)^2} + 1 - \Omega_0\right]^{1/2}$$

We currently know that approximatey $\Omega_{m,0} = 0.3$, $\Omega_{rad,0} = 8 \times 10^{-5}$, and $\Omega_{\Lambda,0} = 0.7$. But since ρ_m, ρ_{rel} depend differently on R from each other and Λ, the balance of these evolves with time as R changes. Initially photons were dominant, then matter, and finally "dark" vacuum energy. (see plots above)

Since radiation is well constrained and only important at very early times. The state and fate of the universe is generally shown in a sort of "phase diagram" for Ω_m and Ω_Λ. Note that these are the *current* values of these factors.

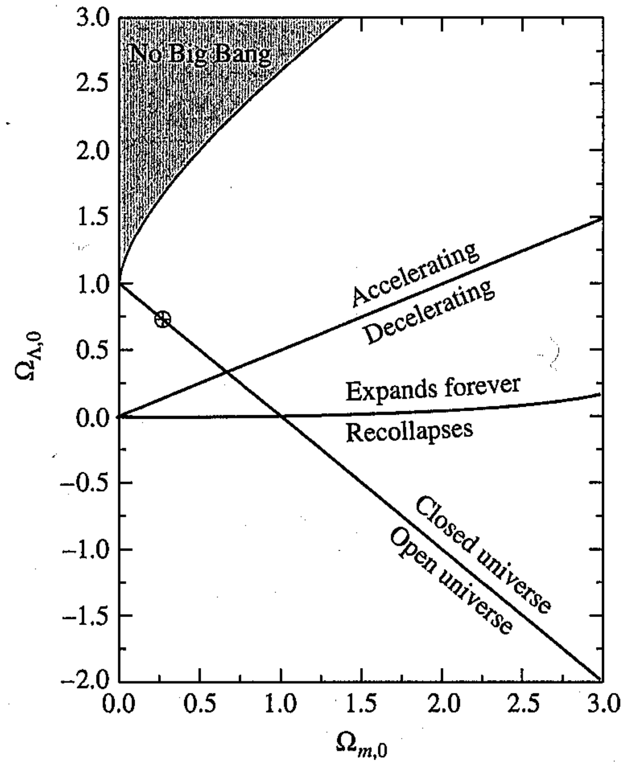

FIGURE 29.21 Model universes on the $\Omega_{m,0}$–$\Omega_{\Lambda,0}$ plane. Every point on this plane represents a possible universe. The point ($\Omega_{m,0} = 0.27$, $\Omega_{\Lambda,0} = 0.73$) is indicated by the circle.

Can see that with no cosmological constant, the expanion is always declerating and the open/closed boundary and the expands forever/recollapses boundary are the same point, $\Omega_{m,0} = 1$. (note open/closed here actually means negatively or positively curved, the naming is a bit archaic. A positively curved universe *might* loop around and "close" like a sphere, but it also may not. It is possible to define an infinite, positively curved 2D surface, it is just not possible to embed such a surface in 3D.)