37 Astro notes 2018/11/28 - Wed - Cosmology - curvature, parameters

37.1 the Hubble parameter in relativistic cosmology

Note that the current expansion rate of the universe is measured by the hubble constant H_0. This appeared in the relation, for any galaxy,

$$v_0 = H_0 d_0$$

where the 0 subscript indicates evaluated at the current time. More generally the expansion rate of the universe is measured by what is called the "Hubble parameter" defined similarly

$$v(t) = H(t)d(t)$$

where the distance for a fixed co-moving coordinate is

$$d(t) = R(t)r$$

Also the recession velocity for the same co-moving coordinate is
(student:)

$$v = \frac{dd}{dt} = \frac{dR}{dt} r$$

And thus

$$v(t) = \frac{dR}{dt} r = \frac{dR}{dt} \frac{d(t)}{R(t)} = H(t)d(t)$$

so that we can identify

$$H(t) = \frac{1}{R(t)} \frac{dR(t)}{dt} = \frac{\dot{R}}{R}$$

Also if we choose $R(t_0) = 1$ we see that H_0 is just the current rate of expansion $H_0 = \dot{R}(t_0)$.

37.2 Curvature

The metric presented above is spatially flat. While this appears to be how our universe is to good accuracy based on observations of the cosmic microwave background, we should consider what it would mean for the universe to be non-flat, i.e. curved.

First it is good to keep in mind that curvature may not mean what you think it means. The best example of this is that while a sphere has positive curvature, a cylinder is flat. (you can turn a flat piece of paper into a cylinder).

Curvature is related to whether distances along certain related paths compare. For example if you draw a "circle" by following a path equidistant from some point in space, how does the length of the path compare to the integrated distance from the "central" point. We will use this to demonstrate different kinds of curvature:

Positive curvature: like in a sphere. The circumference of the path is shorter than it would be for a circle of the same radial distance. This can be seen by consideringa the circumference of a line of lattitude θ on a sphere which is given by 2π times the distance to the axis through the poles. Whereas the distance from the pole to this latitude is the longer distance from the pole down along the surface of the sphere

(draw diagram 29.16)

Negative curvature: is like a saddle point. because the circle is not flat but is flexed upward and downward it is actually longer in circumference than a flat circle. This is like a gathered piece of fabric that has had an extra panel sewn into it - the edge is longer than 2π times its radius, and therefore connot be laid flat.

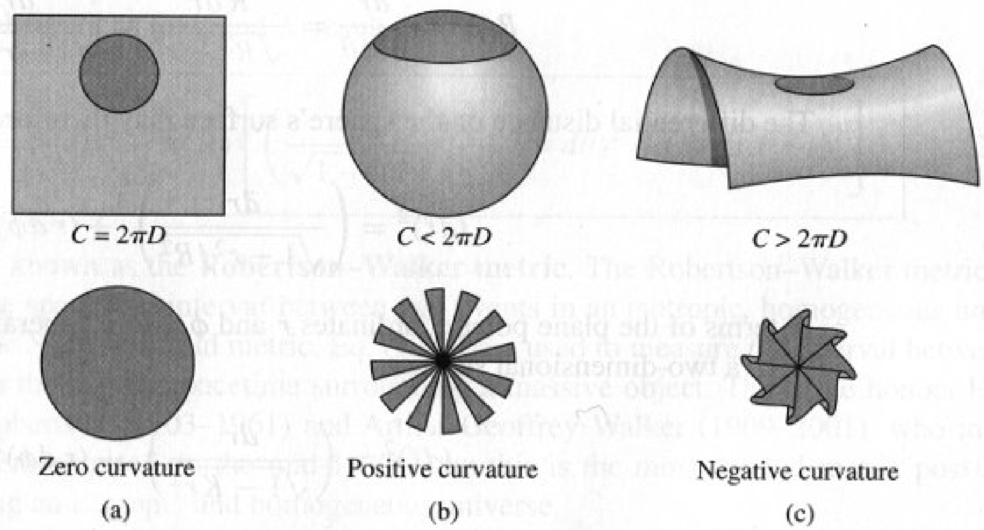

FIGURE 29.17 Calculating the curvature of a surface in three geometries: (a) a flat plane, (b) the surface of a sphere, and (c) the surface of a hyperboloid.

It is possible to add curvature to our metric by adding a term to the radial distance:

$$ds^2 = c^2dt^2 - R^2(t)\left[\left(\frac{dr}{\sqrt{1-kr^2}}\right)^2 + (rd\theta)^2 + (r\sin\theta d\phi)^2\right]$$

This is called the **Robertson-Walker Metric** and is the most general metric that describes a homogeneous and isotropic universe. This therefore essentially sets the way in which we can write down the structure of the universe, and now we must determine R and k for our universe.

37.3 Dynamics of the expansion factor

Einstein's equation (which I will not discuss directly) specifies the dynamical relationship between the spacetime metric and the mass-energy contained in the spacetime. This is the analog of Newton's gravity because it uses the mass-energy configuration to determine the geodesics that free-floating particles will follow. For the Robertson-walker metric and a uniform mass-energy density, the result of Einstein's equations is

$$\left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho - \frac{1}{3}\Lambda c^2\right] R^2 = -kc^2$$

where ρ is the mass-energy density and Λ is the so-called cosmological constant, also called dark energy. This is the Friedmann equation plus the cosmological constant term.

It is customary, and a good idea at early times, to write the density as $\rho_m + \rho_{rel}$ where ρ_{rel} is the mass-energy density of relativistic particles like photons or neutrinos.

Note that

$$\rho_m \propto R^{-3}$$

$$\rho_{rel} \propto R^{-4}$$

$$\rho_\Lambda = constant$$