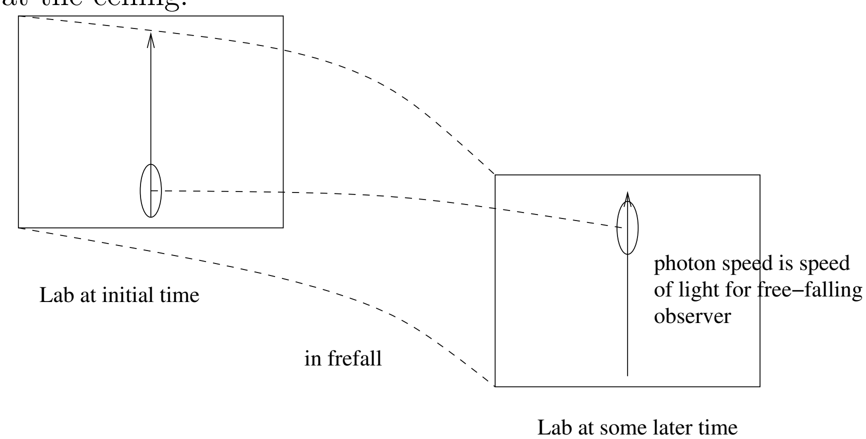
24 Astro notes 2018/10/19 - Fri - General relativity

Last time we saw that an observer outside a freely falling laboratory will see the path of the light curved. This is also consistent with **the light follows the shortest path in both coordinate systems (frames)** in their integrated history. The free-floating observer sees the the photon move properly in the local flat space by moving with it, while the external observer sees the photon find the shortest path due to the curved space that is also causing the free-floating lab to accelerate.

24.1 Effects on light - gravitational redshift

The postulate that a free-falling frame is inertial can also be used to derive the gravitational redshift. Consider a laboratory in free-fall that emits a photon from the floor and detects it at the ceiling.



For the laboratory to be an inertial frame, the observer must not see a shift in the frequency. However, from the point-of-view of the frame in which the observatory is in free-fall, on detection the observer has gained a downward speed of $v = gt = gh/c$ where h is the height the photon is allowed to travel in the lab. In this case the Doppler blue shift should be
(student)

$$\left(\frac{\Delta\nu}{\nu_0}\right)_{observer\ \ motion} = \frac{v}{c} = \frac{gh}{c^2}$$

But, since the free-falling frame is known to be inertial, there must be a compensating effect of gravitational redshift

$$\left(\frac{\Delta\nu}{\nu_0}\right)_{grav} = -\frac{gh}{c^2}$$

so that, for the lab observer, $\Delta\nu/\nu_0 = 0$. Thus photons crossing small distances in a gravitational field are redshifted if they are moving upward. The actual expression for light escaping from a gravitational field is

$$\frac{\nu_\infty}{\nu_0} = \left(1 - \frac{2GM}{r_0 c^2}\right)^{1/2}$$

where r_0 is the radius at which emission occurs, where ν is ν_0.

But where does this arise from? Clocks tick at a different rate different depths within a gravitational field.

$$\frac{\Delta t_0}{\Delta t_\infty} = \left(1 - \frac{2GM}{r_0 c^2}\right)^{1/2}$$

This is called gravitational time dilation and has been measure extensively. $\Delta t_0 < \Delta t_\infty$.

24.2 Structuring the structure of spacetime

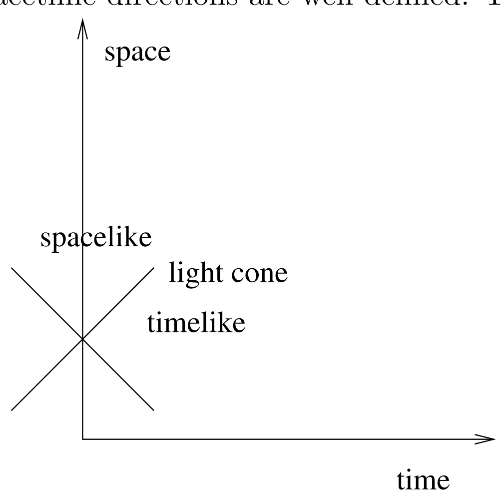
Coordinates vs. spacetime: Before general relativity, it has so far been essentially possible to use coordinates themselves directly to compute things like intervals. For example time differences are simply $t_2 - t_1$.

However, already in special relativity this has become less true. We discussed this as how things were "measured by different observers" but a better way to think about this is that the result of any given "observation" must be computed, and sometimes the coordinates (time for example) in which that result are computed are not the same as the measured analog of that coordinate (e.g. time). For example, using proper intervals (see below), the time interval experienced on a moving spaceship can be computed in the solar system's frame, but it is not the same as the time coordinate in the frame used to compute it.

This drives us to separate coordinate systems from physical intervals, which is closer to reality. Then physical things are generally computed by integrating over a path, rather than differencing coordinates. This is a manifestation of the fact that spacetime is inherently *local*.

Flat space is that special circumstance in which intervals can be measured by differencing for things "at rest" in that frame.

So, if we have given up on space and time coordinate systems what do we have left? Space and time directions! At each point in spacetime, which we usually call an "event", spacetime directions are well defined. This is called the light-cone:



Local spacetime directions are then measured with respect to the light cone, with space-like directions lying outside the light cone and time-like ones within it. Causality, critical for physics, flows strictly within the lightcone.