23 Astro notes 2018/10/17 - Wed - Intro to general relativity

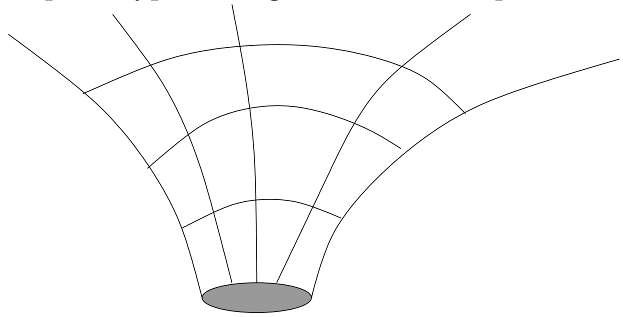
-A problem: excess precession of the perihelion of Mercury
-A question: is light bent by gravity if it has no mass?

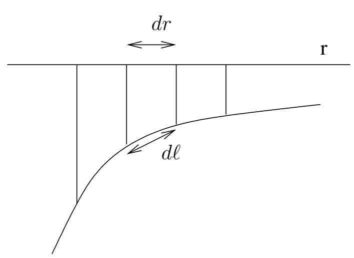
When starting special relativity: had to give up that there is some preferred true frame of reference. But though we said the critical thing was the (local) propagation speed of light, we still defined relation between global coordinate systems moving at some speed. Really need to give up on global (flat) coordinate systems and discuss spacetime by stitching together its local structure.

i.e. the reference frames we used for special relativity are only effectively valid locally, and if we want to relate ones that are far apart we must use general relativity to stitch together what is between them.

What is curved spacetime? The distance between points depends on path, and objects move on "shortest" path, like straight-line motion in flat space.

Explain typical diagram of curved space:





radius can be measured two ways: distance from center, and circumference/2π. The typical curved spacetime diagram is intended to indicate that these are not the same thing i.e. if we measure one radius as

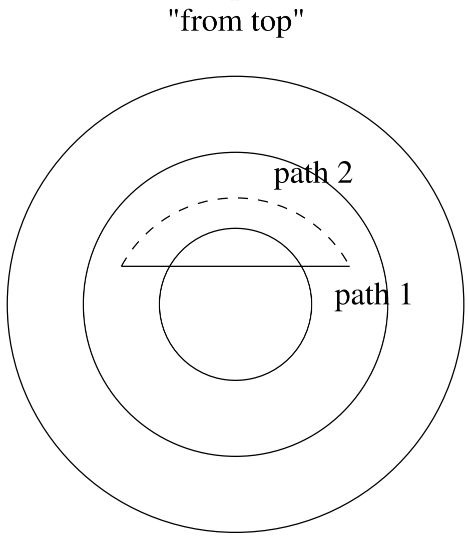
$$r_1 = \frac{s_1}{2\pi}$$

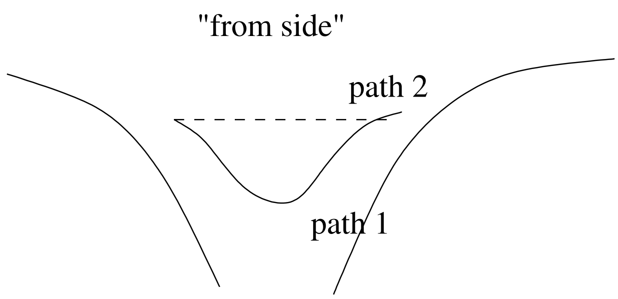
where s_1 is the circumference of some circle about the central object. Then if we move inward by some distance $d\ell$, along the spacetime surface, we would find that if we measure the circumference there s_2 we would find that

$$r_2 = \frac{s_2}{2\pi} \neq r_1 - d\ell$$

since as can be seen from the diagram, the length along the spacetime is longer than the dr between the radii.

This also means what might appear to be a "shortcut" path can actually be longer, so that a normal orbit-like path is the shortest path i.e. the path of an un-forced particle.





Can see that this might change elliptical orbits, for example causing them to not close, since the measurement of distances is no longer the same in the radial and non-radial directions.

In reality things are more complex than this, since we have not drawn the time direction, but the "shortest path" is really the through the 4-dimensional spacetime, not just through space.

23.1 Principle of equivalence

While it is fairly clear that frames moving at constant speed are inertial, i.e. physics works as expected in them, things are less clear for frames subject to the force of gravity. One oddity is that the gravitational mass (i.e. the gravitation "charge") and inertial mass appear to be identical to extremely high accuracy. That is compare the coulomb force

(student)

$$m_i a = -\frac{qQ}{r^2}$$

and the gravitational force

$$m_i a = -\frac{G m_g M}{r^2}$$

it appears that the ratio of m_g, the gravitational "charge", and m_i the inertial mass is constant. In this case

$$a = -\frac{GM}{r^2}$$

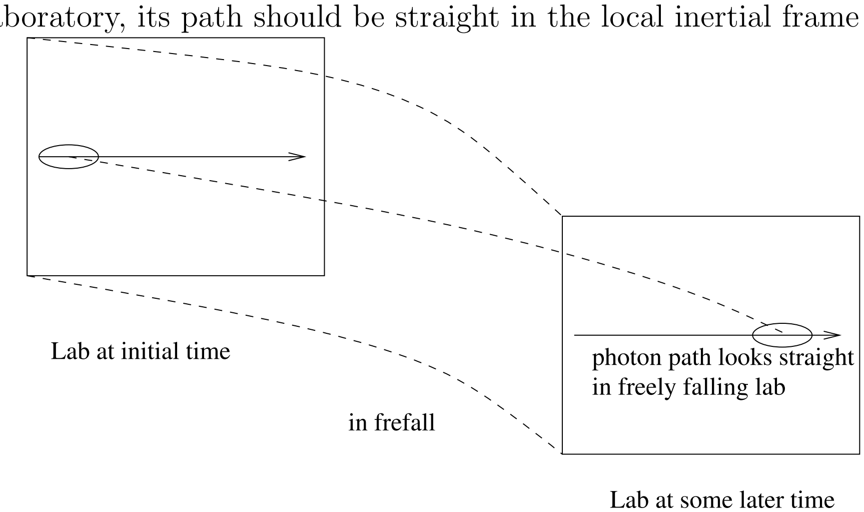
So how can there every be an inertial frame if all frames feel acceleration due to gravity?

The solution to this conundrum is that all objects do feel the same acceleration absolutely – i.e. assume that m_i **and** m_g **are not different things.** This means that all objects in a small region will feel the same a and therefore will move together. i.e. a frame free in a gravitational field acts inertial despite being accelerated.

This principle forms a similar role in general relativity that the constancy of the speed of light does in special relativity. It is the postulate from which we derive the expected behavior of experiments.

23.2 Effects on light - bending

The first effect to consider is the bending of light. If a photon crosses a freely falling laboratory, its path should be straight in the local inertial frame of the laboratory.



As such, an observer outside the laboratory will see the path of the light curved. This is also consistent with **both observers seeing the light follow the shortest path** in their integrated history. The free-floating observer sees the the photon move properly in the local flat space by moving with it, while the external observer sees the photon find the shortest path due to the curved space that is also causing the free-floating lab to accelerate.

We will consider gravitational redshift next time.