22 Astro notes 2018/10/15 - Mon - End of life of stars

22.1 Drivers of stellar evolution

Two drivers for Stellar evolution:

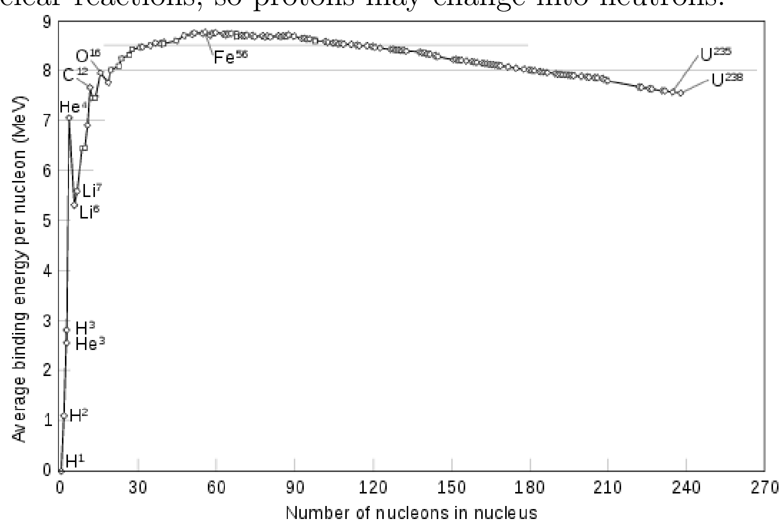
- As a star loses energy, its central temperature will increase.
  (i.e. it has a negative heat capacity!)
  This starts consecutive burning stages. This is halted if the star becomes degenerate – typically the case for low-mass stars.
  Also higher temperature, as in shell burning, leads to higher luminosity.

- Each successive fuel stage releases less energy per mass than the previous.
  While converting H to He releases about 7 MeV per nucleon, later stages release no more than 1 MeV per nucleon each.

More bound means less rest mass energy. So a helium has less rest mass than 2 protons and 2 neutrons. The extra is released when the bound helium is made, and is called the "Binding Energy".

$$E_{\text{bind}} = Zm_pc^2 + Nm_nc^2 - m_ic^2$$

Where m_p is the proton mass, m_n is the neutron mass, and m_i is the mass of the nuclide being considered. Also this is usually given as E_{bind}/A, i.e. "per nucleon", which is what is shown in this plot. This is appropriate because the number of nucleons is conserved during nuclear reactions, so protons may change into neutrons.



(image from binding energy curve entry on wikipedia, but also appearing in many textbooks.)

22.2 H-depleted core formation

$M > M_\odot$: Convective core, burning via CNO cycle (H to He), has a radiative envelope.
$M < M_\odot$: radiative core, pp burning

Kippenhahn diagram. Can see that only the inner regions get burned. also the burning shuts off when the core is converted to helium, which is at 5.6 on the time axis.

Kippenhahn + Weigert

$5 M_\odot$

$1.3 M_\odot$

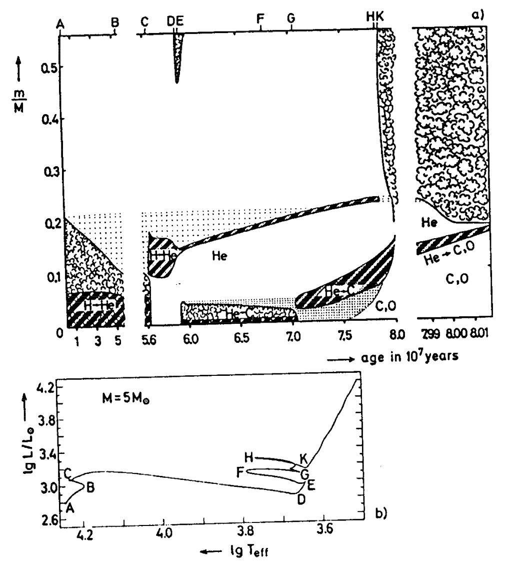


Fig. 31.2. (a) The evolution of the internal structure of a star of $5M_\odot$ of extreme population I. The abscissa gives the age after the ignition of hydrogen in units of 10^7 years; each vertical line corresponds to a model at a given time. The different layers are characterized by their values of m/M. "Cloudy" regions indicate convective areas. Heavily hatched regions indicate where the nuclear energy generation (ε_H or ε_{He}) exceeds 10^3 erg g^{-1} s^{-1}. Regions of variable chemical composition are dotted. The letters $A \dots K$ above the upper abscissa indicate the corresponding points in the evolutionary track, which is plotted in Fig. 31.2 (b). (After KIPPENHAHN et al., 1965)

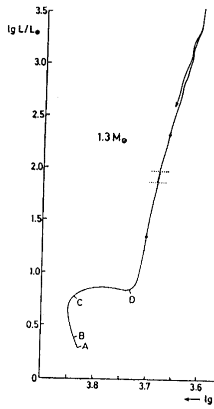


Fig. 32.2. The evolutionary track of a star of $1.3M_\odot$ with the initial composition $X_H = 0.9$, $X_{He} = 0.099$, $Z = 1 - X_H - X_{He} = 0.001$ as computed by THOMAS (1967). The letters $A - D$ refer to the corresponding evolutionary states in Fig. 32.3. The arrows indicate the direction of the evolution. This direction is reversed for a short period between the dotted horizontal lines. This transient drop in luminosity at about $\lg L/L_\odot = 2$ occurs when the hydrogen-burning shell crosses the chemical discontinuity left behind when the bottom of the outer convective zone moves outwards again in the mass scale after it has reached its deepest extension (see Fig. 32.3)

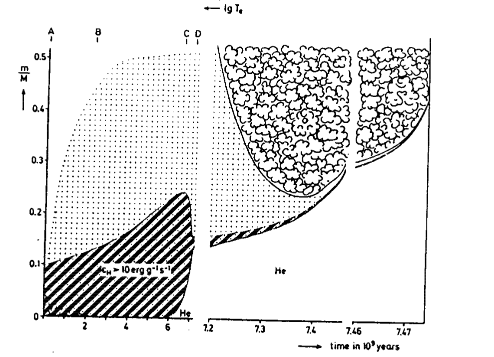


Fig. 32.3. The evolution of the internal structure of a star of $1.3M_\odot$ plotted in the same manner as in Fig. 31.2(a). The main region of hydrogen burning is hatched, "cloudy" areas indicate convection. Regions of variable hydrogen content are dotted. (After THOMAS, 1967)

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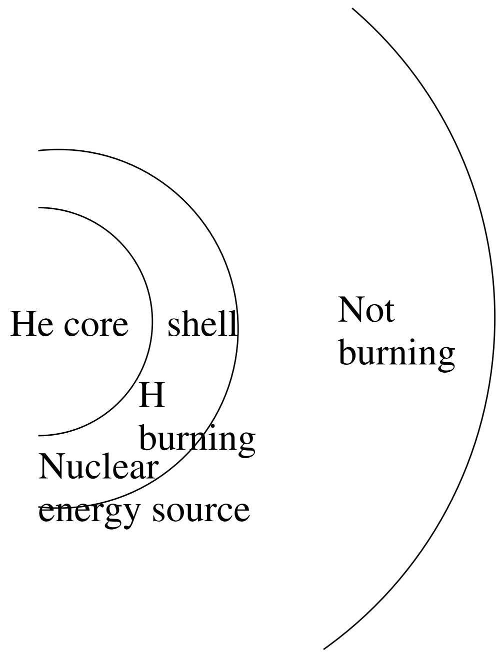
$M > M_\odot$ first. Eventually the temperature does rise as X decreases until the fuel runs out in the core. When this happens the He core gravitationally contracts on t_{KH}. The B to C hook in the HR diagram called the Henyey hook is due to this. Contraction halts once hydrogen burning has ignited in a shell surrounding the helium core.

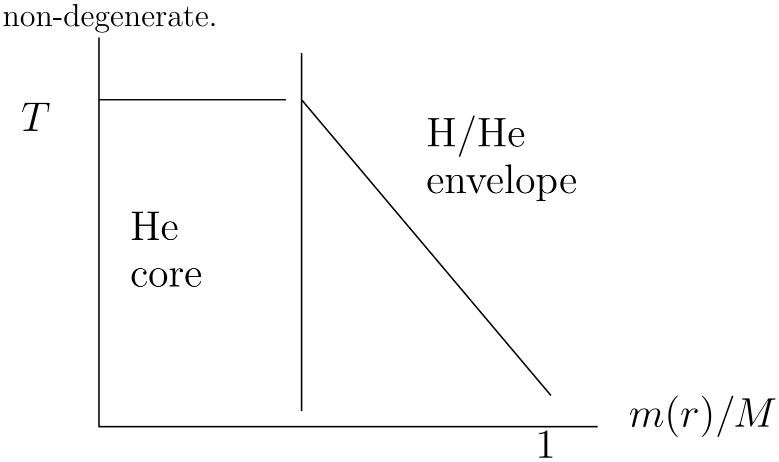
for $M < M_\odot$, T is rising even on the main sequence, so the transition to shell burning is more gradual. Since there is no convective core, there is no sudden depletion, the helium core just slowly grows. Can see this in the Kippenhahn diagrams, as the helium core ramps up at about 6 Gyr.



The helium core mass will be growing in time. Question is can this evolution persist until all H is burned. We will show that the above picture can only be constructed when the helium core is a small fraction of the mass.

22.3 Schönberg-Chandrasekhar Limit

Consider a star after hydrogen burning in the core is done and presume that the He core is non-degenerate.



The envelope exerts pressure on the core and we want to see if there is always a hydrostatic solution. Go back to the virial theorem. Hydrostatic balance:

$$\frac{dP}{dr} = -\rho g = -\frac{Gm(r)\rho(r)}{r^2}$$

multiply both sides by $4\pi r^3$ and integrate (this is what we did to get the virial theorem).

Let R_c be the radius of the core. integrating to here:

$$\int_0^{R_c} 4\pi r^3 \frac{dP}{dr}\, dr = 4\pi r^3 P(r)|_0^{R_c} - 12\pi \int_0^{R_c} r^2 P(r) dr$$

just integrating by parts.

$$= 4\pi R_c^3 P(R_c) - 12\pi \int_0^{R_c} r^2 P(r) dr$$

where now $P(R_c)$ is not zero, since it has the envelope above it. That's the left hand side. The right hand side is the gravitational bining energy which is roughly $-GM_c^2/R_c$. For an isothermal, ideal gas core

$$P(r) = nkT = \left(\frac{\rho}{\mu_c} m_p\right) kT_c = \frac{\rho k T_c}{\mu_c m_p}$$

where μ_c is the mean molecular weight in the core (different from the envelope). Mean molecular weight is approximately the number of protons or neutrons per particle in the (ionized gas). So pure hydrogen has one proton per nucleus and electron so that $\mu = 1/2$, helium has 4 protons and neutrons per 3 particles, one nucleus and two electrons, giving $\mu = 4/3$.

Putting in

$$12\pi \int r^2 \frac{\rho(r) k T_c}{\mu_c m_p} dr = 3 \frac{kT_c}{\mu_c m_p} M_c$$

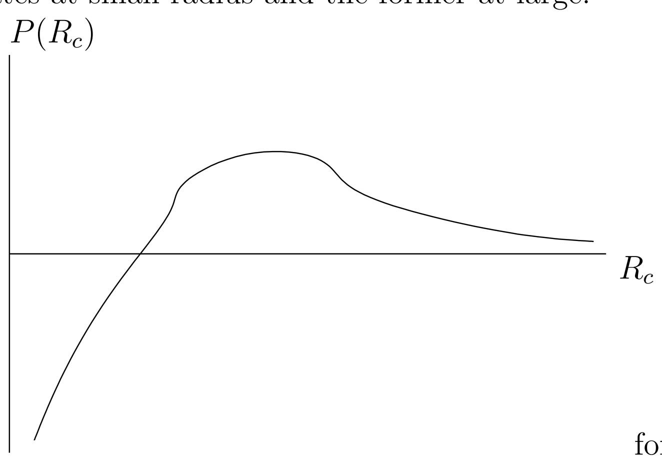
having done all the integrals, we pull it together:

$$4\pi R_c^3 P(R_c) - 3 \frac{kT_c}{\mu_c m_p} M_c = -\frac{GM_c^2}{R_c}$$

Solve for $P(R_c)$,

$$P(R_c) = \frac{3}{4\pi} \frac{kT_c}{\mu_c m_p} \frac{M_c}{R_c^3} - \frac{1}{4\pi} \frac{GM_c^2}{R_c^4}$$

fixing everything and only considering dependence on radius, wee see the latter term dominates at small radius and the former at large.



for fixed M_c, T_c there is a $P_{c,max}$. This gives:

$$R_{c,crit} = \frac{GM_c\mu_c m_p}{kT_c}\frac{4}{9}$$

and

$$P(R_{c,crit}) = \frac{3}{4}\frac{1}{4\pi R_c^3}\frac{M_c}{\mu_c m_p}kT_c = 0.7\left(\frac{kT_c}{\mu_c m_p}\right)^4\frac{1}{G^3M_c^2}$$

(Note the temperature of the core is actually set by the Hydrogen burning shell.)

Now we need to look at the envolope. Pesume that $M = M_{env} \gg M_c$ so that

$$P_{base} = \frac{GM^2}{R^4}$$

what is R here? for the envelope: we would say that

$$kT_e = \frac{GM\mu_e m_p}{R}$$

This is that the hydrogen burning must be distributed over a scale height in order for it to be stable (for the thermostat mechanism of star to work). So then

$$P_{base} = \left(\frac{kT_e}{\mu_e m_p}\right)^4\frac{1}{G^3M^2}$$

To have a solution, we must have $P_{base} < P_{c,max}$. If $T_e = T_c$, then this constraint is just concerned with the relative masses and relative mean molecular weights. (don't believe any coefficients in this derivation) for the scaling we get:

$$\frac{1}{\mu_e^4M^2} < \frac{1}{\mu_c^4M_c^2}$$

or

$$\frac{M_c}{M} < \left(\frac{\mu_e}{\mu_c}\right)^2(0.4)$$

for a stable solution. Here the 0.4 comes from a numerical analysis. Recall that approximately $\mu_e = 0.6$ and $\mu_c = \mu_{He} = 1.33$ and we get

$$\frac{M_c}{M} < 0.08$$

If the Helium core is non-degenerate, then $M_c < 0.08M$ for a stable hydrostatic solution.

Note that this derivation assumes non-degenerate gas – with degeneracy, any pressure can be supported.

Stars that finish core H burning with a core larger than this limit can proceed almost directly to core He burning instead of needing to build the He core more with shell burning first. As a result, some don't even make it to the giant branch before He burning starts.

So we have found the limit for an isothermal He core to support an overlying star:

$$\frac{M_c}{M} < 0.08$$

Above this core contracts directly until He fusion begins. That changes the constant-T core into one with a T gradient, which can be stable.

22.4 With Degenerate core

When the helium core becomes degenerate, then ANY M_c/M value is possible. Won't derive this.

22.5 Helium ignition and the core limit

Three ranges, $M/M_\odot > 6$, $6 > M/M_\odot > 2$ and $2 > M/M_\odot$.

Consider $M > 6M_\odot$. $M_c/M > 0.08$ when the H shell burning ignites. No isothermal solution is possible. Proceeds directly to helium core burning, with no shell phase in between. (that is after burning of H runs out the star "discovers" that it can't support itself, and begins to contract until He ignites) The He core contracts over a time set by the overall energy loss-rate. This is about the Kelvin-Helmholtz time,

$$t = \frac{GM^2/R}{L} \simeq 10^6 \text{ yr at } 6M_\odot$$

or 3×10^4 yr at $30M_\odot$. Refer to first figure of interior of $5M_\odot$ star on Kippenhan figure for borderline case. From C to D is the expansion of the giant, which happens quickly.

This short-lived phase when the He core is contracting is referred to as the Hertzprung Gap. For higher mass stars core burning ignites before reaching giant branch.

Can be seen in upper MS HR diagram, He burning starts at E:

Table 26.7

EVOLUTIONARY PHASES IN FIG. 26.10 AND FIG. 26.11

Phase	Segment of Track or Point(s)	Discussed in Sect.
MS	A–B	26.4a
Central H exhaustion	B–C	26.4b
H burning shell source	C–E	26.4c
Deep convective envelope	D, K	26.4c
Core He burning	E–F	26.4d
Central He exhaustion	F–G	26.4e
He burning shell source	G–H–K	26.4f
Cessation of H burning shell source	H	26.4g

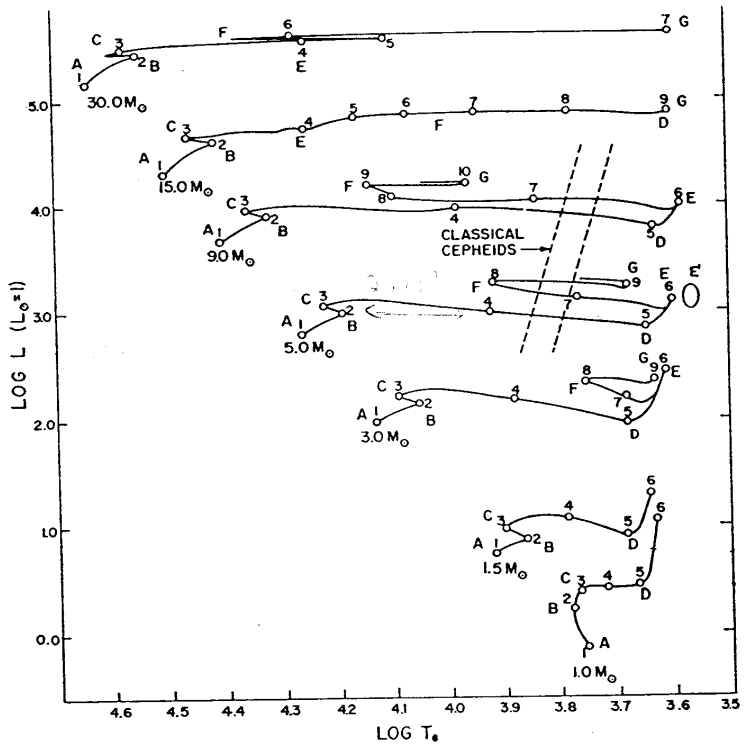


Fig. 26.10 Post-MS evolutionary tracks on the theoretical H-R diagram for stars having masses in the range $1.0 \leq M \leq 30.0$ and, initially, a "Population I" composition ($X = 0.708$, $Z = 0.02$ for all tracks except the $M = 30$ track; for this track, $X = 0.70$, $Z = 0.03$). Numbered points along the tracks are those listed in Table 26.2. The significance of the letters alongside the tracks is summarized in Table 26.7 (*cf.* also Fig. 26.12 below). The tracks for $1.0 \leq M \leq 15.0$ are due to Iben [Ib64]; the track for $M = 30.0$ is due to Stothers [St66d]. The oval labelled E' is the region where the "helium flash" occurs in stars with $M \approx 1.0-1.3$ (see Sect. 26.4c). The approximate location of the classical cepheids is also shown (see Chap. 27).

EVOLUTIONARY LIFETIMES (YEARS)†

(*Initial composition:* $X = 0.708$, $Z = 0.02$ *for* $1.0 \leq M \leq 15.0$; $X = 0.70$, $Z = 0.03$ *for* $M = 30.0$)

	M (solar units)						
Point	1.0	1.5	3.0	5.0	9.0	15.0	30.0
1	5.016(7)	1.821(7)	2.510(6)	5.760(5)	1.511(5)	6.160(4)	2 (4)
2	8.060(9)	1.567(9)	2.273(8)	6.549(7)	2.129(7)	1.023(7)	4.82(6)
3	9.705(9)	1.652(9)	2.394(8)	6.823(7)	2.190(7)	1.048(7)	4.91(6)
4	1.0236(10)	2.036(9)	2.478(8)	7.019(7)	2.208(7)	1.050(7)	4.92(6)
5	1.0446(10)	2.105(9)	2.488(8)	7.035(7)	2.213(7)	1.149(7)	4.93(6)
6	1.0875(10)	2.263(9)	2.531(8)	7.084(7)	2.214(7)	1.196(7)	5.45(6)
7	–	–	2.887(8)	7.844(7)	2.273(7)	1.210(7)	5.46(6)
8	–	–	3.095(8)	8.524(7)	2.315(7)	1.213(7)	–
9	–	–	3.262(8)	8.782(7)	2.574(7)	1.214(7)	–
10	–	–	–	–	2.623(7)	–	–

* Numbers in parentheses are the powers of ten by which the corresponding entries are to be multiplied.

† From Iben [Ib64] and Stothers [St66d].

Now consider $2 < M < 6$. Then the initial value of $M_c/M < 0.08$ when H burning in core is completed. Core can be isothermal during H shell burning, and does not collapse until the H shell burning has forced $M_c > 0.08M$. This happens before the core becomes degenerate. $5M_\odot$ case is example, but core building phase is brief.

Finally $M < 2M_\odot$. Core becomes degenerate before $M_c > 0.08M$. Thus the He core can get as large as it wants until the He ignites. (ends up igniting when it is about $0.4M_\odot$)

22.6 Evolution in density-temperature

final fate of the star is determined by where the curves for burning intersect the degeneracy curves. Refer to plots that show the calculated evolution in this diagram.

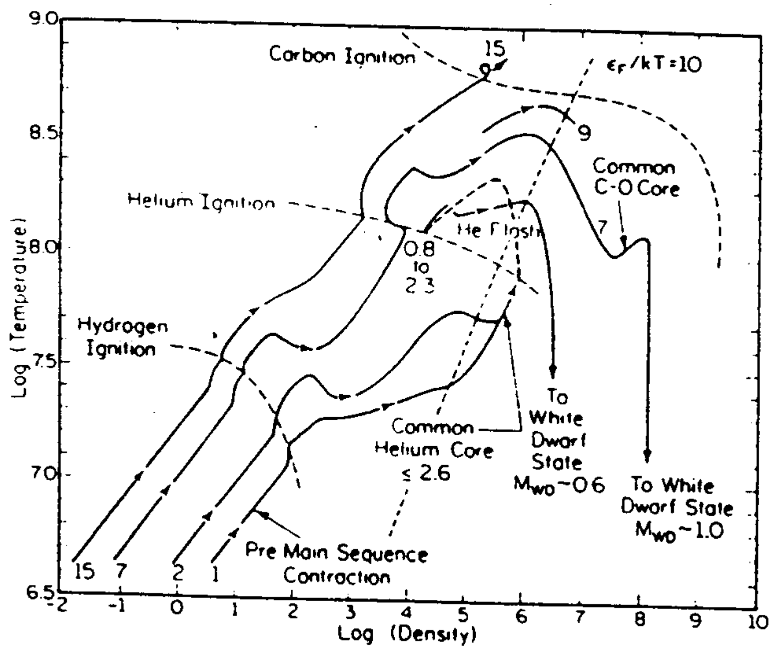


FIGURE 2.7. Central density versus central temperature for evolving stellar mod- els. Reproduced, with permission, from I. Iben Jr. 1985, "The Life and Times an Intermediate Mass Star," in *Quarterly Journal of the Royal Astronomic* Society, Volume 26, published by Blackwell Scientific Publications.

Also similar figures in MESA paper - Figure 22, 29, 30

22.7 Post-Main-Sequence Evolution

Two important effects - Shell burning and Mass loss

Shell burning leads to giant - on lower main sequence ($\lesssim 2M_\odot$) 2 separate giant branches separated by core helium burning. (Horizontal Branch) Shell burning luminosity depends on the mass of the underlying core, not the total mass. Monotonically increasing.

Mass loss important for both low and high mass stars. For low mass stars Riemers law, calibrated on observations:

$$\dot{M} = -4 \times 10^{-13} \eta \frac{L}{gR} M_{\odot}/yr = -4 \times 10^{-13} \eta \frac{L}{GM/R} M_{\odot}/yr$$

where η is an adjustable parameter calibrated on observations.

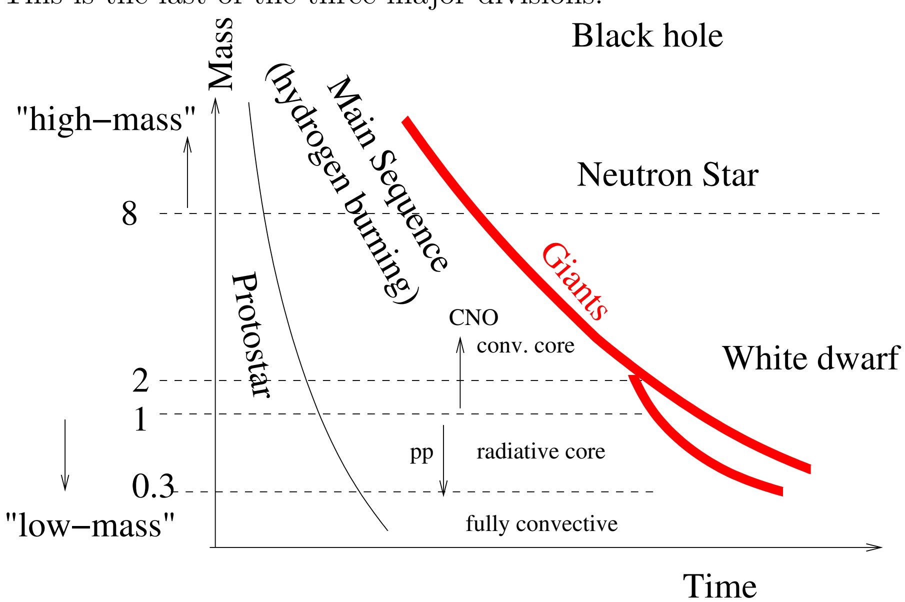
(show movie of low mass star to white dwarf) indicate horizontal branch - only for low mass stars. Mass loss a random variable that gives spread in HB. Was thought to happen during He flash, but now though to just be at tip of RGB.

22.8 end of life

Low mass stars lose mass and end up as white dwarfs. This also forms a planetary nebula.

High mass stars eventually form Fe, which cannot be fused to release energy. The Fe core eventually becomes high enough mass that central pressure causes electron capture $p \rightarrow n$. This removes electron pressure support in a runaway process called core collapse that forms a neutron star. Bounce and neutrinos released powers supernova. (Mesa fig 31)

This is the last of the three major divisions.



22.9 Cluster Color-lum diagrams

Since, in clusters, all stars form at same time, but bright, hot stars evolve in shorter time, the cluster can be dated by distribution of stars in the color-magnitude diagram.

Figures from text:

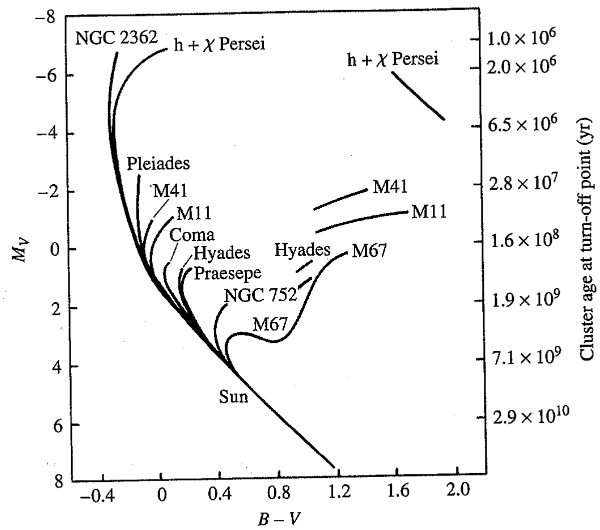


FIGURE 13.19 A composite color–magnitude diagram for a set of Population I galactic clusters

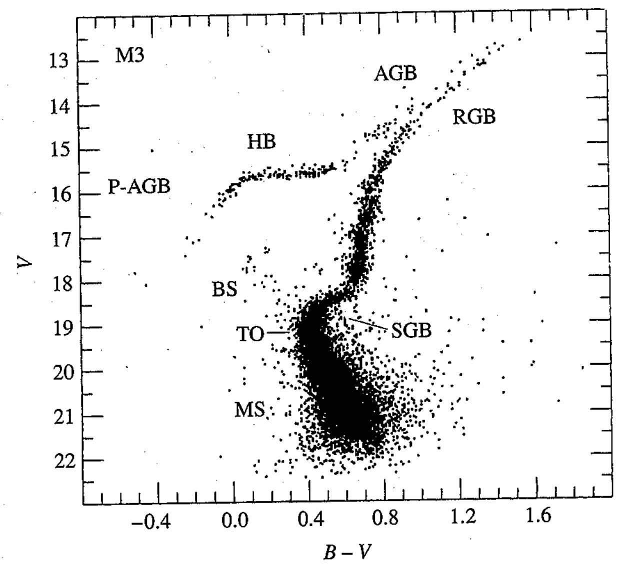


FIGURE 13.17 A color–magnitude diagram for M3, an old globular cluster. The ma