8 Astro notes 2018/9/10 - Mon - special relativity

8.1 Summary

we obtained the Lorentz transformation, the relation between the coordinates of events in a reference frame and one moving at speed u with respect to it in the x direction.

$$t' = \gamma(t - ux/c^2)$$

$$x' = \gamma(-ut + x)$$

$$y' = y, \qquad z' = z$$

where $\gamma = 1/\sqrt{1 - u^2/c^2}$, and note that $\gamma \geq 1$ always.

We will also make use of the inverse transforms which are most easily obtained by swapping u to $-u$ and the primes to non-primes

$$t = \gamma(t' + ux'/c^2)$$

$$x = \gamma(ut' + x')$$

8.2 Time dilation

Consider the ticking of a clock that is in a moving reference frame. We can compute the time between ticks as determined by an observer that is not moving with the clock (note this is an "inverse" transformation from prime to non-prime, so we negate u):

$$t_2 - t_1 = \gamma[(t'_2 - t'_1) + (x'_2 - x'_1)u/c^2]$$

or, since the object is assumed to be at $x' = 0$ at both times,

$$\Delta t_{\text{moving}} = \gamma \Delta t_{\text{rest}}$$

where here "moving" is considered with respect to the clock being considered (i.e. an "at rest" frame is moving with respect to a moving clock).

Thus it appears to take longer, from the point of view of the observer not moving with the clock, for the a tick to elapse than it does in the clocks rest frame.

Note that this is not a "apparent" effect. If the ticks are measured by measuring pulses of light, the discrepancy is that present after accounting for the light travel time.

8.3 Length Contraction

Now consider a bar with ends at x'_2 and x'_1 such that $L' = x'_2 - x'_1$. The lack of simultaneity makes the definition of "measuring" this bar's length in a moving reference frame non-trivial. It must be defined in terms of events. We will take its length in the non-prime frame (which is moving with respect to the bar) to be the distance between the positions that the ends of the bar pass at the same time t.

$$x_2 - x_1 = \gamma(u(t'_2 - t'_1) + x'_2 - x'_1) = \gamma^2[u(t_2 - t_1) - (x_2 - x_1)u^2/c^2] + \gamma(x'_2 - x'_1)$$

since $t_2 - t_1 = 0$, by our assumption,

$$(1 + \gamma^2 u^2/c^2)(x_2 - x_1) = \gamma(x'_2 - x'_1)$$

The left hand side of this is

$$\frac{1 - u^2/c^2 + u^2/c^2}{1 - u^2/c^2}(x_2 - x_1) = \gamma^2(x_2 - x_1)$$

so

$$\gamma^2(x_2 - x_1) = \gamma(x'_2 - x'_1)$$

or

$$L = L'\sqrt{1 - u^2/c^2}$$

where L' is the actual length of the bar in its own rest frame, and L is the length perceived by a moving observer.

8.4 Homework problem on simultaneity (or, rather, lack thereof)

Consider a spaceship that is traveling past the solar system toward a star 10 light-years away at $0.5c$. A signal is emitted from Earth when the spaceship is halfway there, and therefore received just as the spaceship reaches the star.

Draw, to scale as much as possible, a spacetime diagram in both the solar system rest frame and a frame moving with the spaceship that shows the path of the Sun, the distant star, the spaceship and the signal. How far does the signal travel in the frame of the spaceship? By working back from the time the signal was received using the speed of light and the varying distance to the solar system, what fraction of the distance to the star would the occupants of the spaceship say that they had covered at the time, according to their coordinates, that the signal was emitted? (That is, what is the time coordinate of the emission event in the spaceship's frame, and how does this compare to the time coordinate of the spaceship arriving at the star?)

You should explicitly give the coordinates in both the solar system and spaceship frames of the followin events:

- Departure of spaceship from solar system
- Arrival of spaceship at star
- Emission of signal
- Arrival of signal at star

You must also compute and show the positions of both the solar system and the distant star at the beginning and end of the trip in both frames. Verify by comparing the distance and time between the emission and arrival events of the light in the spaceship frame that the light is moving at the speed of light.