6 Astro notes 2018/9/5 - Wed - colors, special relativity

6.1 Filters and color

We have been assuming the wavelength response of our detector is performing the following integral of our flux field

$$F_{\mathrm{bol}} = \int_0^\infty F_\lambda d\lambda$$

This corresponds to measuring the bolometric flux, the energy at all wavelengths. But real detectors generally only measure over a limited range of wavelengths. This is partially *on purpose* because it allows one to get a gross idea of the spectral shape (and therefore the temperature) by just measuring the fux in a few wavelength ranges. So for a real instrument we do something like:

$$F_X = \int_0^\infty F_\lambda(\lambda)\, \mathcal{S}_X(\lambda)\, d\lambda$$

where $\mathcal{S}$ is the sensitivity and X is some label for the "pass band".

We usually select a **filter** that is constructed to reproduce, in combination with our detector, one of several standard sensitivity functions. (Though typically only approximately.)

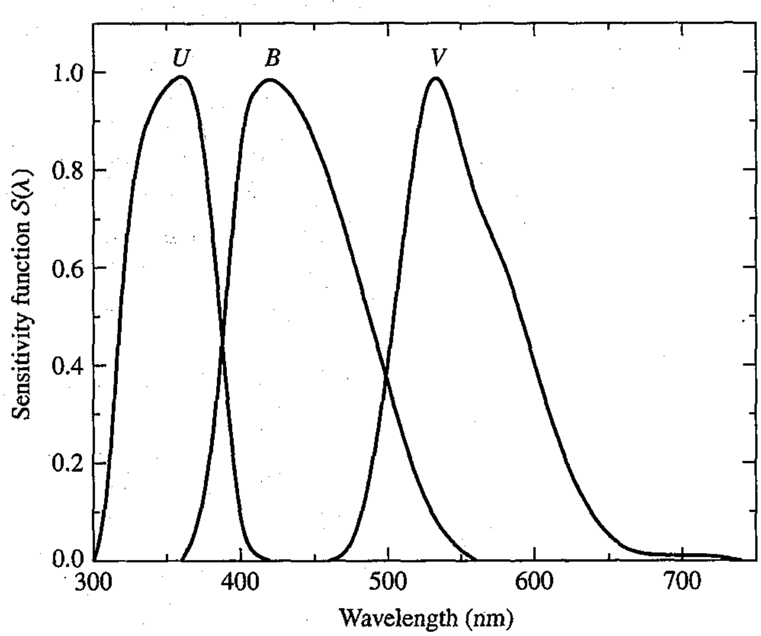

FIGURE 3.10 Sensitivity functions $\mathcal{S}(\lambda)$ for U, B, and V filters. (Data from Johnson, *Ap. J.*, *141*, 923, 1965.)

Filter	center λ (nm)	width $\Delta\lambda$ (nm)
U	365	68
B	440	98
V	550	89

There are also many others, R, I, J, etc and entire other systems, e.g. *ugriz*, which are those from the Sloan Digital Sky Survey (SDSS).

The magnitudes are define generally like so:

$$V = -2.5\log_{10}\left(\frac{\int F_\lambda \mathcal{S}_V d\lambda}{\int F_{\lambda,\mathrm{Vega}} \mathcal{S}_V d\lambda}\right)$$

Where here the star Vega is used as a reference. (not quite really, but that is the historical reference. Now often "AB" magnitudes, based on single-frequency standards.) And similarly for B, V, etc. Thus Vega has $V = B = U = 0$ by definition, though it is certainly not equally bright in all these wavelengths.

A VERY rough way to characterize the spectral shape is the flux ratio obtained effectively by differencing two magnitudes:

$$B - V = -2.5\log_{10}\left(\frac{\int F_\lambda \mathcal{S}_B d\lambda}{\int F_\lambda \mathcal{S}_V d\lambda}\right) + 2.5\log_{10}\left(\frac{\int F_{\lambda,\mathrm{Vega}} \mathcal{S}_B d\lambda}{\int F_{\lambda,\mathrm{Vega}} \mathcal{S}_V d\lambda}\right)$$

$$= -2.5\log_{10}\left(\frac{\int F_\lambda \mathcal{S}_B d\lambda}{\int F_\lambda \mathcal{S}_V d\lambda}\right) + C_{B-V}$$

Also called the **color index**. Here the constant C is set again by the reference flux distribution, that of the star Vega. So this just characterizes the ratio between fluxes at different wavelengths, compared to that ratio in Vega. Approximately:

$$B - V \simeq -2.5\log_{10}\left(\frac{F_\lambda(\lambda_{B,center})\Delta\lambda_B}{F_\lambda(\lambda_{V,center})\Delta\lambda_V}\right) + C_{B-V}$$

which is a simple way to estimate color index.

Note that, due to the negative, larger $B - V$ means that F_B is smaller than F_V i.e. the object is redder.

Note that if the spectrum is characterized by a single parameter, like T for a thermal (or T_{eff} for a stellar) spectrom, there is a simple mapping from color index to that parameter.

Mapping from color index $B - V$ to T_{eff} for luminosity class V stars (main sequence stars). (From Binney & Merrifield, *Galactic Astronomy*, tables 3.7 and 3.10.)

spectral type	$B-V$	$T_{\rm eff}$ (K)
O3	-0.33	52,500
O5	-0.33	44,500
O8	-0.32	35,500
B0	-0.30	30,000
B3	-0.20	18,700
B5	-0.17	15,400
B8	-0.11	11,900
A0	-0.02	9,520
A5	0.15	8,200
F0	0.30	7,200
F5	0.44	6,440
G0	0.58	6,030
G5	0.68	5,770
K0	0.81	5,250
K5	1.15	4,350
M0	1.40	3,850
M5	1.64	3240

Why no change at high T? in Rayleigh-Jeans tail, so ratio indepdent of T

6.2 Discussion of homework

Some discussion of homework questions, particularly problem 4 on homework 1a.