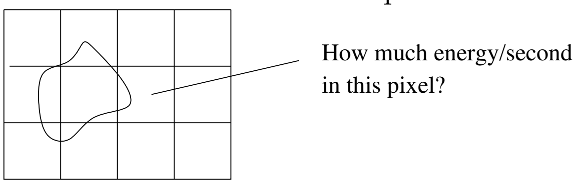
4 Astro notes 2018/8/29 - Wed - Light 2 - EM radiation, general flux density

One way to consider the question we will be addressing is: how do we compute the energy received from a cloud that fills part of a CCD pixel?



Similarly if we are observing a star, the scattered light from the sky will contribute to the energy measured. How do we quantify this? It will be uniform over the sky, but how much ends up in a single pixel in addition to the starlight?

4.1 Electromagnetic radiation

Electromagnetic radiation follows from the equations for electrodynamics. i.e. Maxwell's equations

$$\vec{\nabla} \cdot \vec{E} = 4\pi\rho, \qquad \vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$
$$\vec{\nabla} \cdot B = 0, \qquad \vec{\nabla} \times \vec{B} = \frac{1}{c}\left(4\pi\vec{J} + \frac{\partial E}{\partial t}\right)$$

where ρ is the charge density and $\vec{J}$ is the current density. Note this is Gaussian units (cgs). Note force on a charge then given by

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

For Gaussian units, charge is measured in "electrostatic units" (statcoulombs) rather than Coulombs. This causes a different constant to appear in Gauss' Law (the first equation above) and therefore the typical equation for the electric field of a point particle. In SI $\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0$.

But we want to investigate the situation in empty space where there are no charges (and therefore no currents), so that $\rho = 0$ and $\vec{J} = 0$. Then we have

$$\vec{\nabla} \cdot \vec{E} = 0 \qquad \vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0, \qquad \vec{\nabla} \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

Taking $\vec{\nabla}\times$ of the second equations, and using the identity

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{X}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{X}) - \nabla^2 \vec{X}$$

gives

$$\nabla \times \nabla \times \vec{E} = \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = \nabla \times (-\frac{1}{c}\frac{\partial B}{\partial t})$$

or

$$\frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} - \nabla^2 \vec{E} = 0$$

and similarly for $\vec{B}$

$$\frac{1}{c^2}\frac{\partial^2 \vec{B}}{\partial t^2} - \nabla^2 \vec{B} = 0$$

These clearly indicate waves moving at speed c through space. Solutions look like $\vec{E}_0 e^{i(\vec{k}\cdot\vec{x}-\omega t)}$. Such that

$$\omega^2 - c^2k^2 = 0, \quad \text{or} \quad c = \lambda\nu$$

where λ is the wavelength ($k = 2\pi/\lambda$) and ν is the frequency in cycles per time ($\omega = 2\pi\nu$).

Note that for $\vec{\nabla} \cdot E = 0$, $\vec{k} \cdot \vec{E} = 0$, so that the waves are **transverse**, and the same holds for $\vec{B}$.

Note also that, $\vec{E}$ and $\vec{B}$, are perpendicular to each other as well.

Many important aspects of light follow from the fact that the above equations for propagating light are linear, constant-coefficient wave equations. This means that any two solutions can form a simple **superposition**, i.e. they can just be added and they continue to form a solution and do not influence each other. So when light waves cross they continue to propagate independently. They can interfere, if measured where they overlap, but each propagates independently.

Solutions at different frequencies, ν, and moving in different directions, $\vec{k}$, are independent.

Thus it is not enough just to know how much radiation there is in some region of space, but also what direction it is moving in, i.e. $\vec{k}$.

4.2 Poynting Vector, actual flux density

A straightforward way to characterize energy transfer of a radiation field is by its Poynting vector (in mks units):

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

This gives both the magnitude and direction of energy transfer. However, this is not uniform in space (due to the wave) so it is typically easier to discuss $\langle\vec{S}\rangle$.

Now that we know that light of different wavelengths and directions is indepentent, and that it makes sense to speak of it carrying energy in some direction through space, we will state the fully **general flux density.** The energy passing through a surface (dA) is

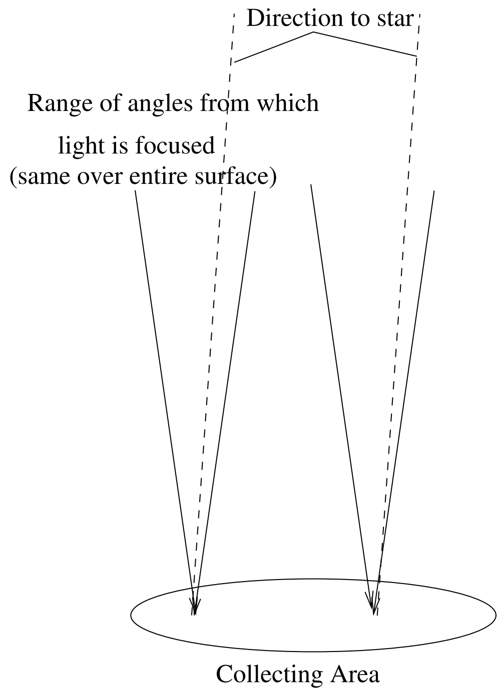
$$dE = F(\lambda, \hat{n}, \vec{x})dAd\lambda d\Omega dt$$

so F has dimensions of energy per wavelength, per solid angle, per area, and depends on wavelength, direction and position. For example, consider a star in space at position $\vec{x}_s$. Far away from the star, its Flux density looks approximately like

$$F = \frac{B(T,\lambda)R^2}{|\vec{x}-\vec{x}_s|^2}\delta(\hat{n} - \widehat{\vec{x}-\vec{x}_s})$$

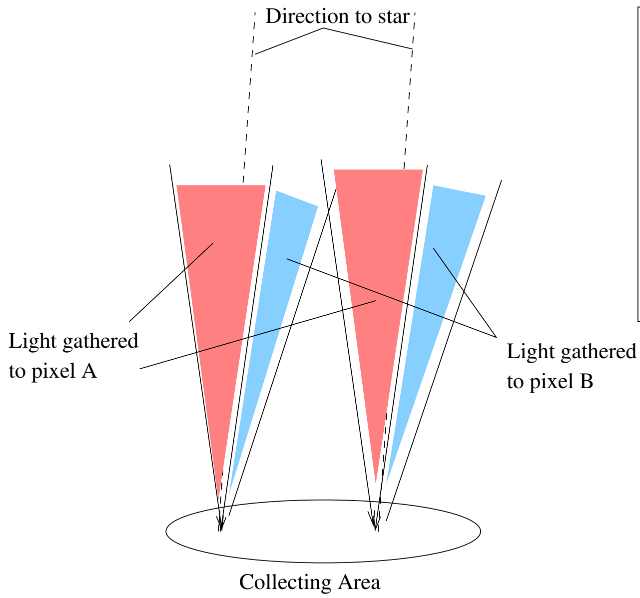
where $B(T,\lambda)$ is the plank function spectrum.

Performing a measurement generally corresponds to integrating F (sometimes called J in this form) over some collecting area *and* over some field of view. The field of view specifies the directionality of photons being collected, and, by design, is very well defined for focusing optics.



For an extended source, the delta function might go away and be replaced by defining F for some value in some portion of the sky. Thus the power measured (energy received per time) is dependent both on how big the collecting area is, and how much of the source light is collected from. If F is approximately independent of position on the sky over some range, then the power collected is just proportional to Ω, the solid angle observed. i.e. the collected flux is the product $F\Omega$. This product having dimensions of energy per time per area per wavelength.

A diagram demonstrating this for neighboring pixes in a detector:


Direction to star
Light gathered to pixel A
Light gathered to pixel B
Collecting Area

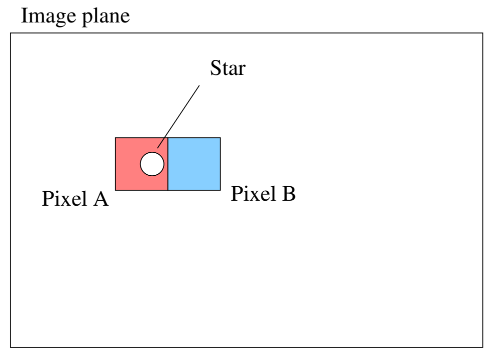


Image plane
Star
Pixel A
Pixel B