1 Astro notes 2018/08/22 - Wed - Celestial Mechanics 1: fundamental equations, limitations

1.1 Intro to course

Topics schedule on syllabus

homework: about 5 homeworks, 2 per each of 3 parts. Talk to each other, but write up individually. Your goal is to communicate to me (or the grader) that you understand the exercise. Less about getting the correct answer than communicating with the grader. Solutions will be handed out soon after due date, so if you have an excuse please let me know right away.

exams: 2 during semester (in-class + take-home) plus final exam

in-class: Will call randomly in class for some exercises and for some homework discussion before HW is due

1.2 Context

The orbit problem formed the basis of the first physics, and arguably is the birth of modern physics, and certainly mechanics.

There are basically 4 stages in the development of our understanding of the orbit problem:

Not physics:
0. Earth-centered motion - clearly the Sun moves across the sky - no, only apparently
1. Kepler's empirical laws - sun-centered - no, center of mass is real center
Physics:
2. Newton's laws - motion about center of mass - no, but we will use this
3. Einstein's laws (General relativity) - orbits don't actually close, light bends despite having no mass, simultaneity isn't well defined, information propogates at finite speed, etc - maybe this is correct (so far it is)

1.3 Fundamental equations for Newton's theory

We wish to consider how the positions of two particles change in time. The first thing we do is construct frame of reference (a rectilinear 3D coordinate system, and a time coordinate):

$$\vec{r}_1(t), \vec{r}_2(t)$$

Newtonian laws of motion and gravity tell us how these behave. The first thing to do is to define an instantaneous state of the objects' motion:

$$\vec{v}_1 \equiv \frac{d\vec{r}_1}{dt}, \quad \vec{v}_2 = \frac{d\vec{r}_2}{dt}$$

Then we introduce the concept of a force and the definition of mass such that

$$\vec{F} = \frac{dm_i v_i}{dt}$$

But Newton said that this is actually nonsense by itself. Forces always come in pairs. In this case there is a force on each object directed towards the other of common magnitude Gm_1m_2/d^2 where d is the separation distance. So we must write two equations to specify the motion:
(ask student)

$$\frac{dm_1v_1}{dt} = \frac{Gm_1m_2}{|\vec{r}_2 - \vec{r}_1|^2}\frac{\vec{r}_2 - \vec{r}_1}{\sqrt{|\vec{r}_2 - \vec{r}_1|^2}}$$
$$\frac{dm_2v_2}{dt} = \frac{Gm_1m_2}{|\vec{r}_2 - \vec{r}_1|^2}\frac{\vec{r}_1 - \vec{r}_2}{\sqrt{|\vec{r}_2 - \vec{r}_1|^2}}$$

At this point we're basically done. These are the equations of motion of the system.

(another student):
Identify: how many functions of time, what are they? how many equations, and what are they? what are initial conditions?

We have 12 time-dependent unknown functions, the components of $\vec{r}_1$, $\vec{r}_2$, v_1, and v_2, and 12 equations, (6 from the definition of $\vec{v}_i$ and 6 from Newton's laws) we just need to set the initial values for the $\vec{r}_i$ and $\vec{v}_i$ and integrate.

Noting some unspoken assumptions...
- Information travels at infinite speed. The position of each object is known when evaluating the force on the other, even though they are separated in space.
- Nominally any reference frame moving at constant velocity is equivalently good.
- Time and space are clearly separated and all simultaneous events are well defined and don't depend on coordinate choice.

All these are actually wrong.

Next time will make equations more convenient...