16 Astro notes 2017/02/17 - Fri - Star Formation - Collapse, Protostars

Some discussion of homework 2 in preparation for Monday's exam.

16.1 Star formation and Jean's mass

Take a piece of the ISM with density ρ and T which has mass M over radius R. the gravitational energy is

$$E_{GR} \sim -\frac{GM^2}{R}$$

Typical densities in a star forming clound are $n \simeq 100/\text{cc}$.

Total KE content of that region is

$$E_{th} \simeq \frac{3}{2}\frac{M}{m_p}kT$$

critical condition (Jeans length or Jeans mass) set by

$$E_{GR} + E_{th} < 0$$

i.e. when cloud is bound. Putting in the energies this is

$$\frac{GM^2}{R} > \frac{3}{2}\frac{M}{m_p}kT$$

Rewrite in terms of density

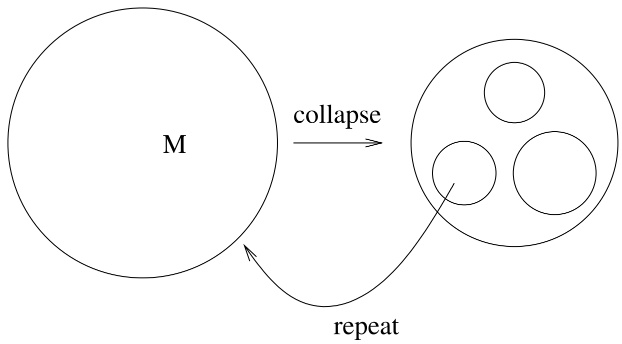
$$M = \frac{4\pi}{3}R^3\rho$$

or

$$M_J = 500M_\odot \left(\frac{T}{10^4\text{ K}}\right)^{3/2}\left(\frac{1\text{ cm}^{-3}}{n}\right)^{1/2}$$

where $n = \rho/m_p$. In a region with T, ρ a mass in excess of this can collapse.

The problem is this is big. How do we get from this to the distribution of stellar masses from this? How does a collapsing mass fragment into $M_\odot$ chunks? We will simply try to answer why would it fragment at all? The Jean's mass scales as $M_J \propto T^{3/2}\rho^{-1/2}$. Imagine that the region keeps it's same temperuture as it collopses. Then M_J decreases during the collapse. Then as the jeans mass decreases this can allow for "fragmentation" or the subsequent collapse of less massive regions.



So what halts the fragmentation. As density is rising in the fragments, they eventually become optically thick and so can't necessarily radiate on the collapse time. After it becomes adiabatic. Then $T \propto \rho^{2/3}$. Then the Jeans mass, $M_J \propto \sqrt{\rho}$, which is increasing as collapse continues. This shuts off fragmentation.

Halting of fragmentation is then due to the isothermal-adiabatic transition. Hard because you have to do both the dynamics and radiative transfer.

What's a characteristic timescale? The dynamical time. This can be obtained by considering a simplified version of free-fall. Consider the acceleration of a particle at the surface of the star if pressure support is removed:

$$\frac{d^2R}{dt^2} = a = -\frac{GM}{R^2}$$

If we use this acceleration as if it were constant we can estimate the time it takes for the particle to cross a distance R:

$$\frac{1}{2}at_{\rm dyn}^2 \approx -R \implies t_{\rm dyn}^2 \simeq \frac{R^3}{GM}$$

One conventional way to write this, which is similar up to factors of order unity, is

$$t_{dyn} = \frac{1}{\sqrt{G\rho}} \simeq \frac{10^7 yrs}{(n/100)^{1/2}}$$

Next time will make a simple estimate of the mass scale at which fragmentation stops.